



The finiteness conjecture for 3×3 binary matrices

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Abstract

The invariant polytope algorithm was a breakthrough in the joint spectral radius computation, allowing to find the exact value of the joint spectral radius for most matrix families [7, 8]. This algorithm found many applications in problems of functional analysis, approximation theory, combinatorics, etc..

In this paper we propose a modification of the invariant polytope algorithm enlarging the class of problems to which it is applicable. Precisely, we introduce mixed numeric and symbolic computations. A further minor modification of augmenting the input set with additional matrices speeds up the algorithm in certain cases.

With this modifications we are able to automatically prove the finiteness conjecture for all pairs of binary 3×3 matrices and sign 2×2 matrices.

1 Introduction

In this paper we are concerned with the maximal asymptotic growth rate of products of matrices, the so called *joint spectral radius*. It has been defined in 1960 [18] and since found applications in many seemingly unconnected areas of mathematics and engineering, e.g. for computing the regularity of wavelets and of subdivision schemes [4], the capacity of codes [16], the stability of linear switched systems [6].

Definition 1.1. Given a finite set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$. The *joint spectral radius* (JSR) of \mathcal{A} is defined as

$$\text{JSR}(\mathcal{A}) := \lim_{n \rightarrow \infty} \max_{A_j \in \mathcal{A}} \|A_{j_n} \cdots A_{j_1}\|^{1/n}, \quad (1)$$

where $\|\cdot\|$ is any sub-multiplicative matrix norm.

An open question in the joint spectral radius theory is the so called *finiteness conjecture* [13]:

Given a matrix set, does there exist a finite product whose powers' spectral radii attain the growth rate equal to its joint spectral radius?

The finiteness conjecture has been proven false, in the sense that such a finite product does not always exist; although this case seems to be exceptional [1, 2, 12, 11, 9]. In this paper we proof the finiteness conjecture for pairs of binary matrices of dimension 3.

1.1 Overview and main results

In Section 2 we present the *invariant polytope algorithm* (ipa) for computing the JSR of a finite set of square matrices. In Section 2.1 we discuss how mixed numeric-symbolic computations can be used to widen the classes of matrices where the ipa is applicable. In Section 2.2 we show how we can augment our input set of matrices to obtain faster termination properties.

Finally, in Section 3 we discuss how the ipa, together with the discussed modifications, can automatically proof the finiteness conjecture for pairs of binary matrices of dimension 2 and 3, as well for pairs of sign matrices of dimension 2.

1.2 Notation

The set of all integers is denoted by \mathbb{N} , integers including zero by \mathbb{N}_0 , reals by \mathbb{R} , non-negative reals by \mathbb{R}_+ , complex numbers by \mathbb{C} . Given $X \subseteq \mathbb{C}^s$, where $s \in \mathbb{N}$ is the dimension, we denote the closure of X by $\text{cl}(X)$ and its interior by X° . Products of sets are understood element wise, e.g. $A \cdot B = \{a \cdot b : a \in A, b \in B\}$. Comparisons of matrices are understood element wise. For a matrix A we denote by A^T its transpose and for a square matrix by $\rho(A)$ its spectral radius.

We will make use of various convex hulls of sets throughout the paper.

Definition 1.2. • For $V \subseteq \mathbb{R}^s$, we define its *convex hull* $\text{co } V$ as the intersection of all convex sets containing V .

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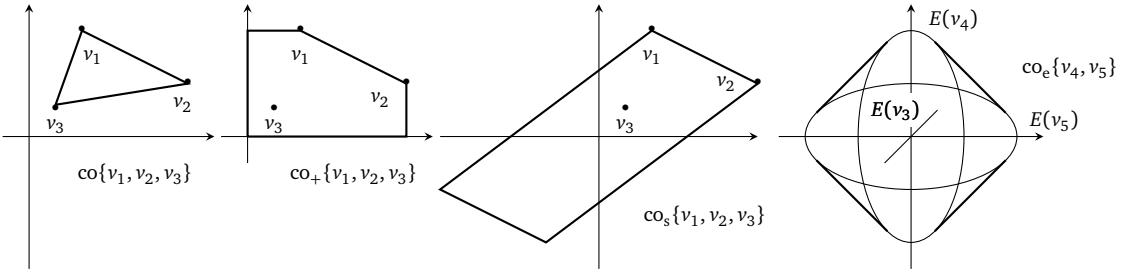


Figure 1: Various convex hulls. $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ i \end{bmatrix}$, $v_5 = \begin{bmatrix} i \\ 2 \end{bmatrix}$.

- For $V \subseteq \mathbb{R}_+^s$, we define the *cone hull* of V (in the first orthant) by

$$\text{co}_+ V = \left\{ x \in \mathbb{R}_+^s : x = y - z, y \in \text{co}(V), z \in \mathbb{R}_+^s \right\} \subseteq \mathbb{R}_+^s. \quad (2)$$

- For $V \subseteq \mathbb{R}^s$, we define the *symmetric convex hull* of V by

$$\text{co}_s V = \text{co}\{V, -V\} \subseteq \mathbb{R}^s. \quad (3)$$

- For $v = a + ib \in \mathbb{C}^s$ we define its corresponding ellipse $E(v) = E(a, b) \subseteq \mathbb{R}^s$ as the two dimensional subset $\{a \cos t + b \sin t : t \in \mathbb{R}\} \subseteq \mathbb{R}^s$. For $V \subseteq \mathbb{C}^s$, we define the *elliptic convex hull* of V by

$$\text{co}_e V = \text{co}\{E(v) : v \in V\} \subseteq \mathbb{R}^s. \quad (4)$$

- For simplicity, we denote with $\text{co}_* V$ any of the convex hulls co_+ , co_s , co_e , depending on the context.

We will use the aforementioned convex hulls to define norms via their unit ball.

Definition 1.3. Let $P \subseteq \mathbb{R}^s$ be a compact, convex set with non-empty interior, and such that $rP \subseteq P$ for all $|r| \leq 1$. We define the *Minkowski norm* $\|\cdot\|_P : \mathbb{R}^s \rightarrow \mathbb{R}$ by

$$\|x\|_P = \min\{r > 0 : x \in rP\}. \quad (5)$$

2 The invariant polytope algorithm

The *invariant polytope algorithm* (*ipa*) for the computation of the JSR makes use of the inequality [4],

$$\max_{A_j \in \mathcal{A}} \rho(A_{j_k} \cdots A_{j_1})^{1/k} \leq \text{JSR}(\mathcal{A}) \leq \max_{A_j \in \mathcal{A}} \|A_{j_k} \cdots A_{j_1}\|^{1/k}, \quad (6)$$

which holds for any $k \in \mathbb{N}$ and any sub-multiplicative norm $\|\cdot\|$. Before we can describe how the ipa works, we need a few further definitions.

Definition 2.1. • For a product $A_{j_k} \cdots A_{j_1}$ we say the number $\rho(A_{j_k} \cdots A_{j_1})^{1/k}$ is its *averaged spectral radius*.

If there exists a product $\Pi = A_{j_n} \cdots A_{j_1}$, $A_j \in \mathcal{A}$, such that $\rho(\Pi)^{1/n} = \text{JSR}(\mathcal{A})$, i.e. its averaged spectral radius equals the joint spectral radius, we call the product a *spectral maximizing product* (*s.m.p.*).

• Given a matrix $\Pi \in \mathbb{R}^{s \times s}$, we call the eigenvalues largest in modulus the *leading eigenvalues* and the corresponding eigenvectors the *leading eigenvectors*. If there exists only one largest eigenvalue in modulus (counted with algebraic multiplicity), we say the leading eigenvalue is *simple*.

• Given a bounded set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$ with $\lambda = \text{JSR}(\mathcal{A}) > 0$ and let $\tilde{\mathcal{A}}$ be the set of normalized matrices $\tilde{\mathcal{A}} = \{A_j / \lambda : A_j \in \mathcal{A}\}$. \mathcal{A} is said to possess a spectral gap (at $\text{JSR}(\mathcal{A})$) if there exists $\gamma < 1$ and for every product $\tilde{\Pi} = \tilde{A}_{j_n} \cdots \tilde{A}_{j_1}$, $\tilde{A}_j \in \tilde{\mathcal{A}}$, which is not an s.m.p., it holds that $\rho(\tilde{\Pi}) < \gamma$.

We are now in the position to describe the ipa, which runs in two stages: Firstly, it guesses spectral maximizing products Π_n , $n = 1, \dots, N$; Secondly, it tries to construct the unit ball P of a vector norm, for whose induced matrix norm all normalized matrices $\tilde{A}_j = A_j / \rho(\Pi_1)^{1/\text{len}(\Pi_1)}$, $A_j \in \mathcal{A}$, have norm less than or equal to 1. If the second part succeeds, then, by Inequality (6), we obtain the exact value of the joint spectral radius.

The construction of the set P is done iteratively. Starting with (properly scaled leading eigenvectors) of the s.m.p.-candidates, in each step it is checked whether all images of all points not yet mapped into the interior (of the convex hull of all formerly computed points) are mapped into the interior (of the convex hull of all formerly computed points). Depending on the structure of the input set, different convex hulls need to be used; *Case (P)*: If all entries of the matrices A_j are non-negative, then we can take non-negative leading eigenvectors of the s.m.p.-candidates as starting vectors and use the cone hull co_+ . *Case (R)*: If the matrices A_j have positive and negative entries and all leading eigenvectors are real, then we use the symmetric convex hull co_s . *Case (C)*: In all other cases we need to use the elliptic convex hull co_e . If eventually all points are mapped into the interior, then an invariant polytope is found and the algorithm terminates.

A simplified pseudo code implementation is given in Algorithm 1. For a more thorough discussion of the algorithm see [7, 8, 14]; For a discussion about the containment problem see [7, 17].

A crucial point in Algorithm 1 is the line `if $r \notin \text{co}_* V^\circ$ then`: If we cannot proof that a vector r is not contained in the *interior* of $\text{co}_* V$, then we have to add it to the set V . Otherwise we would not get rigorous results using this algorithm. Furthermore, with this procedure sufficient and necessary conditions for the termination of the ipa are known.

Algorithm 1: Invariant polytope algorithm

Data: irreducible, finite set of matrices $\mathcal{A} = \{A_j \in \mathbb{R}^{s \times s} : j = 1, \dots, J\}$

Result upon Termination: $\lambda = \text{JSR}(\mathcal{A})$, invariant polytope $\text{co}_* V$

Search for s.m.p.s Π_1, \dots, Π_N , set $\lambda := \rho(\Pi_1)^{1/\|\Pi_1\|}$

Scale matrices $\tilde{\mathcal{A}} := \{\lambda^{-1}A_j : j = 1, \dots, J\}$

Select leading eigenvectors $V := \{v_0, \dots, v_N\}$

Set $R_{\text{new}} := V$

while $R_{\text{new}} \neq \emptyset$ **do**

- Set $R := R_{\text{new}}$
- Set $R_{\text{new}} := \emptyset$
- for** $r \in \tilde{\mathcal{A}}R$ **do**
- if** $r \notin \text{co}_* V^\circ$ **then**
- Set $V := V \cup r$
- Set $R_{\text{new}} := R_{\text{new}} \cup r$

Return $\lambda, \text{co}_* V$

Theorem 2.1 ([8]). Let $\mathcal{A} = \{A_j \in \mathbb{R}^{s \times s} : j = 1, \dots, J\}$ be a finite set of matrices. The ipa terminates if and only if the set \mathcal{A}

- has a spectral gap,
- has only finitely many s.m.p.s $\Pi_n, n = 1, \dots, N$ (up to powers and cyclic permutations), and
- each s.m.p. has only one simple leading eigenvector v_n (up to complex conjugates).

2.1 Mixed numeric/symbolic computations

The conditions on the matrix set \mathcal{A} in Theorem 2.1 sound rather restricting, but it turns out that most matrix families from applications fulfil them. Notable exceptions are when the scaled set $\tilde{\mathcal{A}}$ has a matrix product which is the identity matrix, or when vertices are mapped onto the boundary of the current polytope. In both cases the ipa cannot terminate, since the algorithm always checks whether images of vertices are mapped into the interior of the current polytope.

To overcome these problems one can revert to a symbolic computation of the norm. Unfortunately, a purely symbolic computation is computationally not feasible, because too expensive. Thus, we resort to a mixed numerical and symbolic algorithm to replace the aforementioned line in the algorithm with **if** $s \notin \text{co}_* V$ **then** whenever possible. We distinguish between two cases.

Case 1: Whenever a new vertex point is near to an existing vertex point, we compare their exact coordinates symbolically. This can be done efficiently and just needs some matrix-vector multiplications.

Case 2: Slightly more complicated but still feasible; Whenever the norm of a new vertex is near 1, we compute an exact upper bound of its norm symbolically. This is efficiently possible whenever the leading eigenvectors are all real, i.e. in cases (R) and (P). Indeed, the problem of determining whether a point is inside or outside of a polytope can be stated as an LP problem [7], which does not only answer the containment problem, but also reports the vertices of a face of the polytope through which a ray through the point in question passes. This face can be used to symbolically compute an *upper* bound of the norm. For the case when one leading eigenvector is complex (case (C)) we yet do not have devised an efficient algorithm for the second problem.

Example 2.1 shows how mixed numeric/symbolic computation can be used to solve examples where vertices of the polytope are mapped onto other vertices.

Example 2.1. Let

$$A_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix}.$$

The only s.m.p. of this set is given by A_2 , see below for the proof. The ipa cannot compute the joint spectral radius of this set exactly due to two reasons: (1) The matrix A_2 has multiple leading eigenvalues ± 1 , and furthermore (2), $A_2 = A_2^3$ and thus vertices of any polytope are mapped onto itself after three iterations.

With mixed symbolic and numeric computation we obtain that, with leading eigenvector $v_0 = [\sqrt{2} + 2 \quad \sqrt{2} \quad -2]^T$, the polytope $P = \text{co}_s \{v_0, A_1 v_0, A_1 A_1 v_0, A_2 A_1 v_0, A_2 A_2 A_1 v_0\}$ is invariant under both matrices A_1, A_2 . See Figure 2 for the tree generated by the ipa.

Proof. We prove that A_2 is the only s.m.p. of the set $\{A_1, A_2\}$. First note that $\rho(A_2) = 1$. The claim follows by Gripenberg's algorithm [5]: Since the norm $\|A_1\|_2 = \sqrt{2}/2 < \rho(A_2)$, each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2$ or $\cdots A_2 A_2$, where

$$A_1 A_2^1 = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \text{and} \quad A_2 A_2^1 = \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

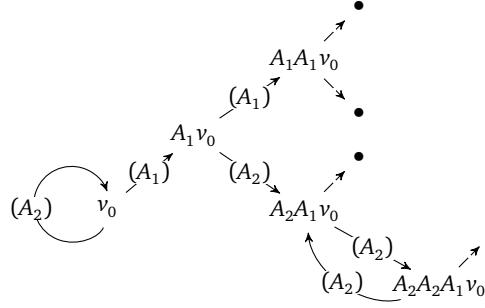


Figure 2: Tree generated by the ipa with mixed numeric/symbolic computations for Example 2.1. The starting vector v_0 is the leading eigenvector of A_2 . Arrows depict how vertices are mapped under the given matrix product. Vertices plotted as \bullet (instead written as text), are mapped to the interior of the polytope $P = \text{co}_* \{v_0, A_1 v_0, A_1 A_1 v_0, A_2 A_1 v_0, A_2 A_2 A_1 v_0\}$.

Again, $\|A_1 A_2^1\|_2 = \sqrt{\frac{\sqrt{5}+3}{8}} \simeq 0.80902 < 1$, and thus each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2^2$ or $\cdots A_2 A_2^2$, where

$$A_1 A_2^2 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad A_2 A_2^2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix}.$$

And again, the norm $\|A_1 A_2^2\|_2 = \sqrt{\frac{\sqrt{5}+3}{8}} \simeq 0.80902 < 1$, and thus each product which is a candidate for an s.m.p. has to start with either $\cdots A_1 A_2^3$ or $\cdots A_2 A_2^3$. Since $A_1 A_2^3 = A_1 A_2^1$ and $A_2 A_2^3 = A_2 A_2^1$ we conclude that all s.m.p. candidates are of the form $A_1 A_2^n$ and $A_2 A_2^n$. The former are no s.m.p.s, since their spectral radii is $1/2$, thus A_2 is the only s.m.p.. \square

2.2 Limit matrices

In some cases it speeds up the computation when one adds matrices to the input set \mathcal{A} in question. In particular, given a set of matrices \mathcal{A} , its joint spectral radius $\text{JSR}(\mathcal{A})$ does not change when elements of the closure $\text{cl } \mathcal{A}$ or its convex hull $\text{co } \mathcal{A}$ (to be understood in the Hausdorff distance using a matrix norm) are added to \mathcal{A} [10, Proposition 1.8],

$$\text{JSR}(\mathcal{A}) = \text{JSR}(\text{cl } \mathcal{A}) = \text{JSR}(\text{co } \mathcal{A}). \quad (7)$$

Lemma 2.2 shows that we may also add limit matrices to the set in question.

Lemma 2.2. *Given matrices $\tilde{\mathcal{A}} \subseteq \mathbb{R}^{s \times s}$ with $\text{JSR}(\tilde{\mathcal{A}}) = 1$; If the ipa terminates for the set $\tilde{\mathcal{A}}$, then the ipa terminates for the set $\mathcal{L} \cup \tilde{\mathcal{A}}$, where \mathcal{L} is the set of all matrices of the limit set of all possible products of $\tilde{\mathcal{A}}$, i.e. of the set $\{\prod_{k=1}^n \tilde{A}_{j_k} : n \in \mathbb{N}, \tilde{A}_j \in \tilde{\mathcal{A}}\}$.*

Proof. If the ipa terminates, then there exists $K \in \mathbb{N}$ such that $\tilde{A}_j v_n \in \text{co}_* V$ for all $\tilde{A}_j \in \tilde{\mathcal{A}}$ with $V = \bigcup_{k=0}^K \tilde{\mathcal{A}}^k \{v_1, \dots, v_N\}$, where $v_n, n = 1, \dots, N$, are the starting vectors for the ipa. Thus, for each $v_n \in V$, there exists $M \in \mathbb{N}$ such that $\tilde{\Pi}_j^m v \in \text{co}_* V$ for all $m \geq M$ and all s.m.p.s $\tilde{\Pi}_j$. In particular, $\tilde{\Pi}_l v \in \text{co}_* V$ for all $\tilde{\Pi}_l \in \mathcal{L}$. \square

Currently we use a heuristic to decide whether to add matrices and which of them. The according rules have not stabilized yet, and thus, we do not report them.

3 The finiteness conjecture

Recalling the definition of a spectral maximizing product (s.m.p.) in Section 2, we make the following definition:

Definition 3.1. A bounded set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$ is said to posses the *finiteness property* if there exists a finite product $\Pi = A_{j_n} \cdots A_{j_1}, A_{j_i} \in \mathcal{A}$ such that $\rho(\Pi)^{1/n} = \text{JSR}(\mathcal{A})$.

As already mentioned in the beginning, it has been shown that there exist sets of matrices such that the normalized spectral radius of every finite product is strictly less than the JSR. In other words, not all sets of matrices posses an s.m.p.. It is an open question whether pairs of *binary matrices* with entries $\{0, 1\}$ or *sign matrices* with entries $\{-1, 0, 1\}$ always posses an s.m.p. [11, 10]. Using the ipa we can check special cases of this question.

Theorem 3.1. *The finiteness conjecture holds for all pairs of*

- (a) *binary matrices of dimension 2* (i.e. with entries $\{0, 1\}$),
- (b) *sign matrices of dimension 2* (i.e. with entries $\{-1, 0, 1\}$), and
- (c) *binary matrices of dimension 3* (i.e. with entries $\{0, 1\}$).

Remark 1. Point 3.1 (a) is already proven in [10, Chapter 4]; Point 3.1 (b) is already proven in [3].

Proof. With our proposed algorithm a proof of (a) and (b) takes some minutes. The proof of 3.1 (c) takes two days (CPU: AMD Ryzen 3600, 6 cores, 64 GB RAM). The used scripts to proof the results can be found at <gitlab.com/tommsch/dolomites> (and/or <tommsch.com/science.php>) and are named `fc_2.m`, `fc_2s.m`, `fc_3.m`. The results in condensed form are tabulated in the Appendix; in more detail they can be found online. \square

Remark 2. If the algorithm would be implemented in a performant language (like C), this approach of checking the finiteness conjecture could also be used for pairs of sign matrices of dimension 3, of which there are approximately 20 million cases to be checked. For larger matrices, this approach is not feasible any more.

3.1 Diminishing the number of cases

To proof 3.1 (c) we have to consider $2^{18} = 262144$ cases (To proof (b) we have to consider $3^8 = 6561$ cases, for (a) $2^8 = 256$ cases). This number can be reduced significantly: For some sets of matrices a concrete s.m.p. is known, other sets share certain symmetries, so that in total we check 15908 cases (For (b) we check 166 cases, for (a) we check 6 cases). We could exploit even more symmetries, but since those are computational hard to check, the total time needed to proof the statement most likely would increase.

Lemma 3.2. *Given $A_1, A_2 \in \mathbb{R}^{s \times s}$; The following pairs have the same joint spectral radius and the finiteness property holds for all or none of them:*

- $\{A_1, A_2\}$,
- $\{\pm A_1, \pm A_2\}$,
- $\{P^T A_1 P, P^T A_2 P\}$ where P is a permutation matrix, and
- $\{S^{-1} A_1 S, S^{-1} A_2 S\}$ where S is an invertible matrix.

Proof. For the proof we use the definition of the joint spectral radius 1.1 with the 2-norm. The statements then follow from the facts that $\|A\|_2 = \|A^T\|_2 = \| -A \|_2$ and $PP^T = SS^{-1} = I$. \square

Lemma 3.3. *Given $A_1, A_2, A_0 \in \mathbb{N}_0^{s \times s}$; If $A_2 \leq A_1$, then $\text{JSR}(\{A_2, A_0\}) \leq \text{JSR}(\{A_1, A_0\})$.*

Proof. For the proof we use the definition of the joint spectral radius 1.1 with the Frobenius norm $\|\cdot\|_F$, and let $X = A_{j_n} \cdots A_{j_1}$, $j_i \in \{2, 0\}$, be a given product. We first construct a new product $\tilde{X} = A_{\tilde{j}_n} \cdots A_{\tilde{j}_1}$, $\tilde{j}_i \in \{1, 0\}$, from X , by replacing all occurrences of A_2 by A_1 . It follows that $\|X\|_F^{1/n} \leq \|\tilde{X}\|_F^{1/n}$, and thus $\text{JSR}(\{A_2, A_0\}) \leq \text{JSR}(\{A_1, A_0\})$. \square

Lemma 3.4. *Given $A_1, A_2 \in \mathbb{N}_0^{s \times s}$; The finiteness property holds whenever $\text{JSR}(\{A_1, A_2\}) \leq 1$.*

Proof. Since the norm of a non-zero integer matrix is always greater than one, it is not possible that the joint spectral radius of a set of integer matrices is strictly between 0 and 1. If $\text{JSR}(\{A_1, A_2\}) = 0$, then clearly both A_1 and A_2 are s.m.p.s. The second case is non trivial and its proof is given in [10, Chapter 3.4]. \square

Corollary 3.5. *Given $A_1, A_2 \in \mathbb{N}_0^{s \times s}$; The finiteness property holds whenever*

- | | |
|--------------------------|----------------------------|
| (a) $A_2 \leq A_1$ | (c) $A_2 \leq I$ |
| (b) $A_1 A_2 \leq A_1^2$ | (d) $A_2 A_1 \leq A_1 A_2$ |

Proof. Again, we use the definition of the joint spectral radius 1.1 with the Frobenius norm $\|\cdot\|_F$, and let $A_{j_n} \cdots A_{j_1}$, $j_i \in \{1, 2\}$, be a given product.

- (a) and (b) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1\|_F^{1/n}$, and thus $\text{JSR}(\{A_1, A_2\}) = \rho(A_1)$
- (c) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1\|_F^{1/\tilde{n}}$ with $\tilde{n} \leq n$, and thus, $\text{JSR}(\{A_1, A_2\}) \leq \rho(A_1)$ which implies $\text{JSR}(\{A_1, A_2\}) = \rho(A_1)$.
- (d) It follows that $\|A_{j_n} \cdots A_{j_1}\|_F^{1/n} \leq \|A_1^{n_1} A_2^{n_2}\|_F^{1/n}$ for some $n_1 + n_2 = n$, and thus, $\text{JSR}(\{A_1, A_2\}) = \max\{\rho(A_1), \rho(A_2)\}$. \square

Lemma 3.6. *If there exists a norm $\|\cdot\|$ such that $\max\{\rho(A_1), \rho(A_2)\} = \max\{\|A_1\|, \|A_2\|\}$, then the finiteness property holds. In particular, the finiteness property holds for sets of normal matrices, and thus, symmetric matrices. A matrix A is normal, iff $A^T A = A A^T$.*

Proof. The first part follows from Inequality (6), which reads for products of length 1 as $\max_{A_j \in \mathcal{A}} \rho(A_j) \leq \text{JSR}(\mathcal{A}) \leq \max_{A_j \in \mathcal{A}} \|A_j\|$. By the assumptions we have equality here, and thus $\text{JSR}(\{A_1, A_2\}) = \max_{A_j \in \mathcal{A}} \rho(A_j)$.

The second parts about normal matrices follows now by using the 2-norm, which equals the matrix' the largest singular value. For normal matrices the largest singular value equals the largest eigenvalue in magnitude, and thus $\max_{A_j \in \mathcal{A}} \rho(A_j) = \max_{A_j \in \mathcal{A}} \|A_j\|_2$. \square

Definition 3.2. *Given a finite set of matrices $\mathcal{A} \subseteq \mathbb{R}^{s \times s}$; If there exists $V \in \mathbb{R}^{s \times s}$ such that $V A_j V^{-1} = \begin{bmatrix} B_j & C_j \\ 0 & D_j \end{bmatrix}$ for all $A_j \in \mathcal{A}$, then \mathcal{A} is reducible.*

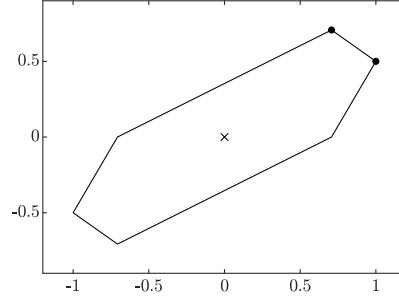


Figure 3: Invariant polytope for the matrices $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$. of Example 4.1. The cross \times denotes the origin, the dots \bullet the leading eigenvectors of the matrices (and s.m.p.s) A_1 and A_2 .

Theorem 3.7. *In the notation from Definition 3.2; If \mathcal{A} is reducible, then $\text{JSR}(\mathcal{A}) = \max\{\text{JSR}(\mathcal{B}), \text{JSR}(\mathcal{D})\}$, $\mathcal{B} = \{B_j : j = 1, \dots, J\}$, $\mathcal{D} = \{D_j : j = 1, \dots, J\}$.*

The first rigorous proof known to the author can be found in [10, Proposition 1.5]. Although the proof is straight forward, it is also rather technical and we abstain from giving it here.

Corollary 3.8. *Given $A_1, A_2 \in \mathbb{Z}^{s \times s}$, $Z \subseteq \mathbb{Z}$ and using the notation from Definition 3.2; The finiteness conjecture holds whenever there exists $S \in \mathbb{C}^{s \times s}$ such that the matrices $S^{-1}A_j S$, have joint block diagonal form with blocks $B_j \in \mathbb{Z}^{s_B \times s_B}$, $D_j \in \mathbb{Z}^{s_D \times s_D}$, $j = 1, 2$, $s_B < l$, and the finites property holds for all pairs of matrices in $\mathbb{Z}^{s_B \times s_B}$.*

Proof. This follows from Lemma 3.2 and Theorem 3.7. \square

4 Implementation notes

Our Matlab implementation of the algorithm can be found on Gitlab [15], and is extensively documented. The file `manual.pdf` gives an overview of the toolbox, in depth documentation can be found directly in the source files, and can be viewed by typing `help functionname` or `edit functionname`, e.g. `help tjsr` or `edit tjsr`, in Matlab.

The main function for the JSR computation is `tjsr`, short for *invarianT polyTope algoriThm*. Depending on the input, our implementation chooses its parameters automatically and usually there is no need for the user to specify options by hand. For example, of the 15910 cases checked for Theorem 3.1 (c), manual intervention was only necessary for 2 cases.

Example 4.1 presents how to use the `tjsr` algorithm.

Example 4.1. Given the matrices $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$. To compute their joint spectral radius, the matrices must be passed as a cell array to the algorithm, e.g. by typing:

```
tjsr( {[0 1;0 1],[1 0;1 -1]} )
```

The algorithm (version 1.2022.05.25) produces the following output:

```
Input: 2 matrices of dimension 2

A lot of candidates found. Nearly all orderings are smp's.
Set <'epseigenplane',inf, 'epsspectralradius',inf, 'maxsmpdepth',5, 'balancing',-1, 'ncdelta',1>.
JSR (of block) 1: 1.0000 1.6180
Duplicate leading eigenvectors occurred. Enable symbolic computation. Set <'epssym',5e-12>.
JSR (of block) 1: 1.0000 1.6180
Case (R).
Selected candidates: | 1 | 2 |
Number of vertices: 2 ( candidates )
Balance 2 Trees. Balancing vector found: [ 1, 341/305]
JSR = [ 1, 1.61803398875 ], norm= Inf, #test: 1/1, #V:2/2 | 0
JSR = [ 1, 1.61803398875 ], norm= 2.41421508763, -
Number of vertices of polytope: 3
Products which give lower bounds of JSR: | 1 |
Algorithm terminated correctly. Exact value found.
JSR = 1
```

One can see that the algorithm restarts two times. The first time because two many s.m.p. candidates are found and appropriate options are set: `<'epseigenplane',inf, 'epsspectralradius',inf, 'maxsmpdepth',5, 'balancing',-1, 'ncdelta',1>`. The second time because the s.m.p. candidates do not have a unique leading eigenvector and mixed symbolic and numeric computation is enabled `<'epssym',5e-12>`.

The third time the algorithm terminates after two iterations. It reports two s.m.p.s A_1 and A_2 , and a joint spectral radius of 1. The constructed polytope with 3 vertices is given in Figure 3. The figure is produced by calling `tjsr` with the option '`plot`': `tjsr([0 1;0 1],[1 0;1 -1], 'plot','polytope')`. A complete list of all options can be found in the file `tjsr_option`, and viewed by typing `tjsr help` or `edit tjsr_option`.

Remark 3. Since our implementation of the algorithm is in Matlab, which has very restricted capabilities for symbolic computations, mixed symbolic computation only works when the leading eigenvectors are expressible in a “simple” closed form, e.g. for integer matrices of dimension less than or equal to 3.

A List of cases

A.1 2×2 binary matrices

The following list reports \mathcal{A} : the set of matrices and *s.m.p.*: the shortest s.m.p. found for the set \mathcal{A} . All unreported cases can be reduced to a simpler one, or an s.m.p. is known due to the structure of the set \mathcal{A} , by the Lemmata presented in Section 3.1.

\mathcal{A}	$s.m.p.$	\mathcal{A}	$s.m.p.$	\mathcal{A}	$s.m.p.$
$\left\{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right\}$	$A_1 A_2^4$	$\left\{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\right\}$	$A_1 A_2^3$	$\left\{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\right\}$	$A_1 A_2$
$\left\{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$	$A_2^2 A_2$	$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right\}$	$A_1 A_2^2$	$\left\{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right\}$	$A_1 A_2$

A.2 2×2 sign matrices

To save space, we use the following abbreviations: + for $+1$, - for -1 , o for 0. In addition to \mathcal{A} and *s.m.p.*, as reported in Section A.1, we mark all cases where at least one leading eigenvalues is complex (case (C)). If the case is not explicitly mentioned, then it is case (R), meaning that all leading eigenvalues are real.

A.3 3×3 binary matrices

Due to the large number of cases and the need to save space, we give the matrices as a pair of two numbers, where the binary representation of the numbers corresponds to the entries in the matrices. The rightmost digit in the binary representation corresponds to the entry in the second matrix, third row, third column. For example, the pair $3/477$, written in the binary system as $000'000'011_2/111'011'101_2$ corresponds to the matrix pair $3/477 \triangleq 000'000'011_2/111'011'101_2 \triangleq \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \right\}$. Matrix pairs with the same s.m.p. are subsumed under one entry. For example, the s.m.p. A_2 of the matrix pair $3/477$ can be found in the section with $\boxed{A_1 = 3}$ and the cell with the entry $\boxed{476 - 78, A_2}$.

A₁ = 2	77	A₂	92	A₁A₂⁵	93	A₂	101	A₂	104	A₁A₂	105	A₁A₂²	108-9	A₂	113	A₁A₂⁵	116	A₁A₂⁶	117	A₂	120	A₁A₂⁵		
121	$A_1A_2^3$	124	$A_1A_2^4$	125	$A_1A_2^2$	141	A_2	156-57	A_2	169	$A_1A_2^5$	172-73	A_2	184-85	A_2	188-89	A_2	204-5	A_2	220-21	A_2	225	$A_1A_2^6$	
228-29	A_2	232	A_2	233	$A_1A_2^2$	236-37	A_2	240-41	A_2	244-45	A_2	248	A_2	249	$A_1A_2^7$	252-53	A_2	332-33	A_2	348-49	A_2	352	A_2	
353	$A_1A_2^5$	356-57	A_2	360	A_2	361	$A_1A_2^4$	364-65	A_2	368	$A_1A_2^5$	369	$A_1A_2^6$	372-73	A_2	376-377	A_2	$A_1A_2^4$	380-81	A_2	396-97	A_2	496-97	A_2
412-13	A_2	424-25	A_2	428-29	A_2	440-41	A_2	444-45	A_2	460-61	A_2	476-77	A_2	480-81	A_2	484-85	A_2	488-89	A_2	492-93	A_2	496-97	A_2	
500-1	A_2	504-5	A_2	508-9	A_2	$A_1 = 3$		76	A_1	77-78	A_2	92	$A_1A_2^2$	93	$A_1A_2^6$	94	A_2	97	$A_1A_2^2$	100	$A_1A_2^2$	101-2	A_2	
104	A_1	105	$A_1A_2^2$	106	A_2	108	$A_1A_2^2$	109-10	A_2	113	$A_1A_2^4$	114	A_2	116	$A_1A_2^5$	117-18	A_2	120-121	A_2	$A_1A_2^2$	122	A_2		
124	$A_1A_2^2$	125	$A_1A_2^2$	126	A_2	141-42	A_2	156-58	A_2	169	A_1A_2	170	A_2	172-74	A_2	184-86	A_2	188-90	A_2	204-6	A_2	220-22	A_2	
225	$A_1A_2^2$	226	A_2	228-30	A_2	232	A_1	233	$A_1A_2^2$	234	A_2	236-38	A_2	240-42	A_2	244-46	A_2	248	A_2	249	$A_1A_2^6$	250	A_2	
252-54	A_2	332-34	A_2	348-50	A_2	352	A_1	353	$A_1A_2^4$	354	A_2	356-58	A_2	360	A_1	361	$A_1A_2^2$	362	A_2	364-66	A_2	424-26	A_2	
368-369	$A_1A_2^5$	370	A_2	372-74	A_2	376	$A_1A_2^4$	377	$A_1A_2^2$	378	A_2	380-82	A_2	396-98	A_2	412-14	A_2	428-30	A_2	500-2	A_2	504-6	A_2	
440-42	A_2	444-46	A_2	460-62	A_2	476-78	A_2	480-82	A_2	484-86	A_2	488-90	A_2	492-94	A_2	496-98	A_2	508-10	A_2					

$A_1 = 6$	72-74	$A_1 A_2$	75	A_2	88	$A_1 A_2$	89	$A_1 A_2^2$	90-91	A_2	92	$A_1 A_2$	93	A_2	97	$A_1 A_2^2$	99-1	A_2	104	$A_1 A_2$					
105	$A_1 A_2^2$	106	$A_1 A_2$	107	A_2	108	$A_1 A_2$	109	A_2	113	$A_1 A_2^3$	114-15	A_2	116	$A_1 A_2^5$	117	A_2	120	$A_1 A_2$						
124	$A_1 A_2$	125	$A_1 A_2^5$	152	$A_1 A_2^2$	153	$A_1 A_2^3$	154-57	A_2	169	$A_1 A_2^4$	170-73	A_2	184	A_2	185	$A_1 A_2^7$	186-89	A_2	122-23	A_2				
218-21	A_2	232	$A_1 A_2$	233	$A_1 A_2^2$	234-35	A_2	240-45	A_2	248	A_2	249	$A_1 A_2^4$	250-53	A_2	344	$A_1 A_2$	345	$A_1 A_2^2$	346-47	A_2				
368-369	$A_1 A_2^4$	370-73	A_2	376	$A_1 A_2^2$	377	$A_1 A_2^3$	378-81	A_2	440-45	A_2	504-7	A_2	$A_1 = 7$		72-75	$A_1 A_2$	78	A_2						
88-89		$A_1 A_2$	90-91	A_2	92-93		$A_1 A_2$	94	A_2	97	$A_1 A_2^2$	99	A_2	100	$A_1 A_2^2$	101-2	A_2	104-106	$A_1 A_2$						
107	A_2	108-109		$A_1 A_2$	110	A_2	113	$A_1 A_2^2$	114-15	A_2	116	$A_1 A_2^5$	117	$A_1 A_2^6$	118	A_2	120-121	$A_1 A_2$	122-23	A_2					
124	$A_1 A_2^2$	125	$A_1 A_2^3$	126	A_2	152-153	$A_1 A_2^2$	154-58	A_2	169	$A_1 A_2^2$	170-74	A_2	184	A_2	185	$A_1 A_2^6$	186-90	A_2						
216-217		$A_1 A_2$	218-22	A_2	232-233		$A_1 A_2$	234-35	A_2	238	A_2	240-46	A_2	248	A_2	249	$A_1 A_2^3$	250-54	A_2						
344-345		$A_1 A_2$	346-47	A_2	350	A_2	368	$A_1 A_2^4$	369	$A_1 A_2^3$	370-74	A_2	376-377		$A_1 A_2^2$	378-79	A_2	380	$A_1 A_2^6$	381-82	A_2				
440-46	A_2	504-7	A_2	510	A_2	$A_1 = 10$		71	A_2	86	$A_1 A_2^2$	87	A_2	99	A_2	101-3	A_2	104	A_1	105	$A_1 A_2^2$	108	A_1		
109	A_2	113	$A_1 A_2^5$	114-15	A_2	116	$A_1 A_2^4$	117-19	A_2	120	$A_1 A_2^2$	121	$A_1 A_2^3$	124	$A_1 A_2^2$	125	$A_1 A_2^7$	126-31	A_2	247	A_2	326-27	A_2		
342-43	A_2	352	A_1	353	$A_1 A_2^5$	354-59	A_2	360	A_1	361	$A_1 A_2^2$	364-65	A_2	368	$A_1 A_2^2$	369	$A_1 A_2^5$	370	A_2	372-75	A_2				
376-377		$A_1 A_2^4$	380-81	A_2	486-87	A_2	503	A_2	$A_1 = 11$		70-71	A_1	86	A_1	87	$A_1 A_2^4$	96-97	A_1	99-2	A_1	103	A_2			
104-6	A_1	108-10	A_1	112-14	A_1	115	A_2	116-18	A_1	119	A_2	120-21	A_1	122	A_2	124	A_1	125	$A_1 A_2^2$	126	A_2	162-67	A_1		
172-74	A_1	178-82	A_1	183	$A_1 A_2^4$	188-90	A_2	230	A_1	231	A_2	246-47	A_2	326	A_1	327	A_2	342	A_1	343	A_2	352-54	A_1		
355	A_2	356	A_1	357-59	A_2	360-62	A_1	364	A_1	365-66	A_2	368-69	A_1	370	A_2	372	A_1	373-75	A_2	376	A_1	377	$A_1 A_2^3$		
378	A_2	380	A_1	381-82	A_2	418-23	A_1	428-30	A_2	434-39	A_2	444-46	A_2	486-87	A_2	502-3	A_2	$A_1 = 12$	67	$A_1 A_2^2$	71	A_2			
75	A_2	83	$A_1 A_2^3$	86	$A_1 A_2^4$	87	A_2	90-91	A_2	99	A_2	102-3	A_2	106-7	A_2	114-15	A_2	118-19	A_2	122-23	A_2	131	$A_1 A_2$		
134-135		$A_1 A_2$	139	A_2	147	$A_1 A_2^3$	149	$A_1 A_2^5$	150	$A_1 A_2^2$	151	$A_1 A_2^3$	152	A_2	153	$A_1 A_2^5$	154-55	A_2	163	$A_1 A_2^6$	165	$A_1 A_2^6$	195	$A_1 A_2^2$	
166	A_2	167	$A_1 A_2^3$	169	$A_1 A_2^6$	170-71	A_2	176-82	A_2	183	$A_1 A_2^2$	184-87	A_2	193	$A_1 A_2$	194		$A_2 A_1 A_2 A_1 A_2$							
197	A_2	198		$A_1 A_2 A_1 A_2^2$	199	A_2	200-201		$A_1 A_2$	202		$A_1 A_2 A_1 A_2^2$	203	A_2	208	A_2	209	$A_1 A_2$							
210	$A_1 A_2 A_1 A_2^2$	211	$A_1 A_2^2$	212	$A_1 A_2^3$	213	A_2	214	$A_1 A_2^2$	215	$A_1 A_2^5$	216	A_2	217	$A_1 A_2^3$	218-19	A_2	225	$A_1 A_2$	226-32	A_2	226-32	A_2		
233	$A_1 A_2^3$	234-35	A_2	240-51	A_2	323	$A_1 A_2^2$	326-27	A_2	330	$A_1 A_2^2$	331	A_2	338	$A_1 A_2^6$	339	$A_1 A_2^2$	342-43	A_2	346-47	A_2	354-55	A_2		
358-59	A_2	362-63	A_2	370-71	A_2	374-75	A_2	378-79	A_2	386	A_2	387	$A_1 A_2$	388	A_2	389	$A_1 A_2^2$	390	A_2	391	$A_1 A_2^3$	393	$A_1 A_2^5$		
394	$A_1 A_2^4$	395	A_2	400-402		$A_1 A_2^4$	403	$A_1 A_2^2$	404	$A_1 A_2^4$	405	$A_1 A_2^5$	406-407		$A_1 A_2^4$	408	$A_1 A_2^4$	409	$A_1 A_2^5$	410-11	A_2				
416-27	A_2	432-43	A_2	449	$A_1 A_2^3$	450		$A_1 A_2^2 A_1 A_2^1 A_2$	451	$A_1 A_2^5$	452-55	A_2	456	$A_1 A_2^2$	457	$A_1 A_2^3$	458	$A_1 A_2^2$	459	$A_1 A_2^5$	464	$A_1 A_2^4$	474	$A_1 A_2^4$	
465	$A_1 A_2^2$	466	$A_2 A_1 A_2^4$	467	A_2	468-71	A_2	472-473		$A_1 A_2^4$	474-75	A_2	480-88	A_2	489	$A_1 A_2^5$	490-92	A_2	494	A_2	496-8	A_2	510	A_2	$A_1 = 13$
66	$A_1 A_2^2$	67	$A_1 A_2$	70	$A_1 A_2^2$	71	A_2	74	$A_1 A_2^2$	75	A_2	78	A_2	83	$A_1 A_2$	86	$A_1 A_2^2$	87	$A_1 A_2^3$	90-91	A_2				
94	A_2	98	$A_1 A_2^2$	99	$A_1 A_2$	102-3	A_2	106-7	A_2	110	A_2	114-15	A_2	118-19	A_2	122-23	A_2	126	A_2	128	A_1	129	$A_1 A_2$		
130	$A_1 A_2$	131	$A_1 A_2$	132	A_1	133	$A_1 A_2$	134	$A_2 A_2^2$	135	$A_1 A_2$	136	A_1	137	$A_1 A_2$	138	$A_1 A_2^2$	139	A_2	140	$A_1 A_2^2$	142	$A_1 A_2^2$		
144	A_1	145	$A_1 A_2$	146		147	$A_1 A_2$	148	$A_1 A_2^2$	149	$A_1 A_2^3$	150	$A_1 A_2^2$	151	$A_1 A_2^2$	152	$A_1 A_2^2$	153	$A_1 A_2^3$	154-56	A_2	158	A_2		
160	A_1	161	$A_1 A_2$	162		163	$A_1 A_2$	164	A_1	165	$A_1 A_2$	166	$A_1 A_2$	167	$A_1 A_2$	168	A_1	169	$A_1 A_2$	170-72	A_2	174	A_2		
176-82	A_2	183	$A_1 A_2^2$	184-88	A_2	190	A_2	192	$A_1 A_2^2$	193	$A_1 A_2$	194	$A_1 A_2$	195	$A_1 A_2$	196	$A_1 A_2$	197	A_2	198	$A_1 A_2^2$	199	$A_1 A_2$		
200	$A_1 A_2$	201	$A_1 A_2$	202	A_2	203	$A_1 A_2$	204	$A_1 A_2^2$	206	A_2	208	$A_1 A_2^2$	209	$A_1 A_2$	210	$A_1 A_2$	211	$A_1 A_2^2$	212	$A_1 A_2^2$	213	$A_1 A_2^2$		
214-215		$A_1 A_2^2$	216	$A_1 A_2^2$	217	$A_1 A_2^2$	218	$A_1 A_2^2$	219	$A_1 A_2^2$	220	$A_1 A_2^2$	221	$A_1 A_2^2$	222	$A_1 A_2^2$	223	$A_1 A_2^2$	224	$A_1 A_2^2$	225	$A_1 A_2^2$			
233	$A_1 A_2$	234-36	A_2	238	A_2	240-52	A_2	254	A_2	255	A_2	256-37	A_2	258	A_2	259-32	A_2	260	A_2	261	A_2	262	A_2		
339	$A_1 A_2 A_1 A_2^2$	342-43	A_2	346-47	A_2	350	A_2	354-55	A_2	358-59	A_2	362-37	A_2	364-48	A_2	368-39	A_2	370-71	A_2	374-75	A_2	378-79	A_2		
382	A_2	384	A_1	385	$A_1 A_2$	386	$A_2 A_2^2$	387	$A_1 A_2$	388	$A_2 A_2^2$	389	$A_1 A_2$	390	$A_2 A_2^2$	391	$A_1 A_2$	392	$A_1 A_2$	393	$A_1 A_2^3$	394	$A_1 A_2^2$		
395	$A_1 A_2^5$	396	A_2	398	A_2	400	$A_1 A_2^3$	401-402	A_2	403	$A_1 A_2^3$	404	$A_1 A_2^3$	405	$A_1 A_2^5$	406	$A_1 A_2^2$	407	$A_1 A_2^2$	408	$A_1 A_2^2$	409	$A_1 A_2^2$		
409	$A_1 A_2^5$	410	$A_1 A_2^2$	411-14	A_2	416-25	A_2	426-30	A_2	432-46	A_2	448-50	A_1	451	$A_1 A_2$	452	A_1	453	A_2	454	A_1	455	A_2		
456	A_1	457	$A_1 A_2$	458	$A_1 A_2^2$	459	$A_1 A_2$																		

200-202	$A_1 A_2$	203	$A_1 A_2^2$	204-7	A_2	216-217	$A_1 A_2$	218-20	A_2	222	A_2	225	$A_1 A_2$	226-31	A_2	232-233	$A_1 A_2$							
234-44	A_2	246	A_2	248	A_2	249	$A_1 A_2^3$	250-52	A_2	254	A_2	330	$A_1 A_2 A_1 A_2^2$	331	$A_1 A_2^4$	334-35	A_2							
354-55	A_2	358-59	A_2	362-63	A_2	366-67	A_2	370-71	A_2	374	A_2	378-79	A_2	382	A_2	393	$A_1 A_2^3$							
408-409		$A_1 A_2^3$		410	$A_1 A_2^8$	411-12	A_2	414	A_2	424-31	A_2	440-44	A_2	446	A_2	456	$A_1 A_2$							
460-63	A_2	472-473			$A_1 A_2^4$	474	$A_1 A_2^4$	475-76	A_2	478	A_2	480-88	A_2	489	$A_1 A_2^4$	490-0	A_2							
$A_1 = 22$		72-74		$A_1 A_2$	75	A_2	76	$A_1 A_2$	77-79	A_2	88-89		$A_1 A_2$	90-91	A_2	92	$A_1 A_2$							
96	A_1	97-98		$A_1 A_2^2$	99	A_2	100	A_1	101-3	A_2	104	$A_1 A_2$	105	$A_1 A_2^2$	106	$A_1 A_2$	107	A_2						
112	$A_1 A_2$	113	$A_1 A_2^3$	114-15	A_2	116	$A_1 A_2^3$	117	A_2	120	$A_1 A_2$	121	$A_1 A_2^2$	122-23	A_2	124	$A_1 A_2$	125	$A_1 A_3$					
139	A_2	140	$A_1 A_2$	141-43	A_2	152	$A_1 A_2$	153	$A_1 A_2$	154-55	A_2	156	$A_1 A_2$	157	A_2	168	$A_1 A_2$	169	$A_1 A_2^2$					
185	$A_1 A_2^4$	186-89	A_2	200-202		$A_1 A_2$	203	A_2	204	$A_1 A_2$	205-7	A_2	216-218		$A_1 A_2$	219	A_2	220	$A_1 A_2$					
224-226		$A_1 A_2^2$	227-31	A_2	232	$A_1 A_2$	233	$A_1 A_2^2$	234-40	A_2	241	$A_1 A_2^6$	242-45	A_2	248	$A_1 A_2$	249	$A_1 A_2^2$						
329	$A_1 A_2^2$	330	$A_1 A_2$	331	$A_1 A_2^6$	332-35	A_2	344	$A_1 A_2$	345	$A_1 A_2^2$	346	$A_1 A_2 A_1 A_2^7$	347-49	A_2	352	$A_1 A_2^2$	353	$A_1 A_2^3$					
360	$A_1 A_2$	361	$A_1 A_2^2$	362-67	A_2	368	$A_1 A_2^3$	369	$A_1 A_2^2$	370-73	A_2	376	$A_1 A_2$	377	$A_1 A_2^2$	378-79	A_2	380	$A_1 A_2 A_1 A_2^7$					
392	$A_1 A_2$	393	$A_1 A_2^3$	394		$A_1 A_2 A_1 A_2^3$	395-99	A_2	408	$A_1 A_2$	409	$A_1 A_2^3$	410	$A_1 A_2$	411-13	A_2	424-31	A_2	440-45	A_2				
457	$A_1 A_2^2$	458	$A_1 A_2$	459	$A_1 A_2^3$	460-63	A_2	472	$A_1 A_2$	473	$A_1 A_2^2$	474	$A_1 A_2 A_1 A_2^3$	475-77	A_2	480	A_2	481	$A_1 A_2^6$					
489	$A_1 A_2^3$	490-1	A_2	504	A_2	505	$A_1 A_2^4$	506-9	A_2	$A_1 = 23$		72	$A_1 A_2^2$	73-75	$A_1 A_2$	76	$A_1 A_2^2$	77-78	$A_1 A_2$					
79	A_2	88-90		$A_1 A_2$	91	A_2	92-93		$A_1 A_2$	94	A_2	96	$A_1 A_2^2$	97	$A_1 A_2^2$	98	$A_1 A_2^2$	99	$A_1 A_2^2$					
101	A_2	102	$A_1 A_2^2$	103	A_2	104	$A_1 A_2^2$	105-106	$A_1 A_2$	107	A_2	108	$A_1 A_2^2$	109	$A_1 A_2$	110-11	A_2	112	$A_1 A_2^2$					
114-15	A_2	116	$A_1 A_2^2$	117	$A_1 A_2^4$	118	A_2	120-121		$A_1 A_2$	122-23	A_2	124	$A_1 A_2^2$	125	$A_1 A_2^2$	126	A_2	136	$A_1 A_2^3$				
137-138		$A_1 A_2$	139	$A_1 A_2^2$	140	$A_1 A_2^2$	141-142		$A_1 A_2$	143	A_2	152-153		$A_1 A_2$	154	$A_1 A_2^2$	155	$A_1 A_2$	156	$A_1 A_2$				
157-58	A_2	168	$A_1 A_2^3$	169	$A_1 A_2$	170-71	A_2	172	$A_1 A_2^3$	173-75	A_2	184	A_2	185	$A_1 A_2^2$	186-90	A_2	200-204	$A_1 A_2$	205	A_2			
206	$A_1 A_2$	207	A_2	216-218		$A_1 A_2$	219	A_2	220	$A_1 A_2$	221-22	A_2	224	$A_1 A_2^2$	225	$A_1 A_2^2$	226	$A_1 A_2^2$	227	A_2	228	$A_1 A_2^2$		
229-31	A_2	232-234		$A_1 A_2$	233-40	A_2	241	$A_1 A_2^4$	242-46	A_2	248	$A_1 A_2$	249	$A_1 A_2^2$	250-54	A_2	328	$A_1 A_2^2$	329-331	$A_1 A_2$				
332-35	A_2	344-346		$A_1 A_2$	347-50	A_2	352	$A_1 A_2^2$	353	$A_1 A_2^2$	354-59	A_2	360	$A_1 A_2^2$	361	$A_1 A_2$	362-67	A_2	368	$A_1 A_2^3$	369	$A_1 A_2^3$		
370-74	A_2	376	$A_1 A_2$	377	$A_1 A_2^2$	378-79	A_2	380		$A_1 A_2 A_1 A_2^3$	381	$A_1 A_2$	381-82	A_2	392	$A_1 A_2^3$	393-394	$A_1 A_2$	395	$A_1 A_2^3$	406-63	A_2		
396-99	A_2	408	$A_1 A_2$	409	$A_1 A_2^2$	410	$A_1 A_2^3$	411-14	A_2	424-31	A_2	440-46	A_2	456-488	$A_1 A_2$	459	$A_1 A_2 A_1 A_2^2$	460-63	A_2	504	A_2	505	$A_1 A_2^3$	
472-474		$A_1 A_2$	475	$A_1 A_2^3$	476-78	A_2	480	A_2	481	$A_1 A_2^4$	482-88	A_2	489	$A_1 A_2^2$	490-2	A_2	504	A_2	505	$A_1 A_2^3$	506-10	A_2		
$A_1 = 27$	70-71	A_1	86-87	A_1	96-97	A_1	99-10	A_1	111	A_2	112-22	A_1	124-26	A_1	125-21	A_1	126-27	A_1	127-28	A_1	128-29	A_1		
352-58	A_1	359	A_2	360-66	A_1	367	A_2	368-70	A_1	372-74	A_1	375	A_2	376-78	A_1	380-81	A_1	382	A_2	383	A_2	384-43	A_1	
$A_1 = 28$	67	$A_1 A_2^2$	71	A_2	74		$A_1 A_2 A_1 A_2^2$	75	A_2	78-79	A_2	82	$A_1 A_2$	83	$A_1 A_2^2$	84	$A_1 A_2$	85	A_2	86-91	A_2			
99	A_2	102-3	A_2	106-7	A_2	111-10	A_2	114-15	A_2	118-19	A_2	122-23	A_2	130	$A_1 A_2^2$	131	$A_1 A_2$	134	$A_1 A_2^2$	135	$A_1 A_2$	136	$A_1 A_2^3$	
137		$A_2 A_2^2 A_1 A_2$	138	$A_1 A_2$	139	A_2	141-43	A_2	144	$A_1 A_2^2$	145	$A_1 A_2$	146	$A_1 A_2^2$	147	$A_1 A_2$	148	$A_1 A_2^2$	149	$A_1 A_2^3$	150	$A_1 A_2^2$		
151	$A_1 A_2^2$	152-153		$A_1 A_2$	154-55	A_2	162	$A_1 A_2^2$	163	$A_1 A_2$	164-165		$A_1 A_2^2$	166	$A_1 A_2^2$	167	$A_1 A_2$	169	$A_1 A_2^3$	170-78	A_2			
179	$A_1 A_2^5$	180-82	A_2	183	$A_1 A_2^4$	184-87	A_2	193	$A_1 A_2$	194	$A_1 A_2^2$	195	$A_1 A_2$	197	A_2	198	$A_1 A_2$	199	A_2	200-202	$A_1 A_2$			
203-7	A_2	208	$A_1 A_2^2$	209	$A_1 A_2$	210		$A_1 A_2 A_1 A_2^2$	211	$A_1 A_2$	212	$A_1 A_2^2$	213	$A_1 A_2$	214	$A_1 A_2^3$	215	$A_1 A_2^3$	216	$A_1 A_2$	217-217	$A_1 A_2$		
218-19	A_2	225		$A_1 A_2 A_1 A_2^3$	226-31	A_2	232	$A_1 A_2$	233		$A_1 A_2 A_1 A_2^2$	234-48	A_2	249	$A_1 A_2^2$	250-51	A_2	253	$A_1 A_2^3$	256-27	A_2			
330	$A_1 A_2^3$	331	$A_1 A_2^6$	334-35	A_2	338	$A_1 A_2$	339	$A_1 A_2^2$	342-43	A_2	346-47	A_2	354-55	A_2	358-59	362-63	A_2	366-67	A_2	370-71	A_2		
374-75		378-79	A_2	386	$A_1 A_2^2$	387	$A_1 A_2^2$	388	$A_1 A_2^2$	389	$A_1 A_2^5$	390	$A_1 A_2^2$	391	$A_1 A_2^3$	393	$A_1 A_2^4$	394	$A_1 A_2$	395-99	A_2			
400-401		$A_1 A_2^2$	402	$A_1 A_2^2$	403	$A_1 A_2^2$	404	$A_1 A_2^2$	405	$A_1 A_2^2$	406	$A_1 A_2^2$	407-409	$A_1 A_2^2$	410	$A_1 A_2^2$	411-21	A_2	416	$A_1 A_2^2$	417-218	$A_1 A_2$		
449	$A_1 A_2^3$	450	$A_1 A_2^2$	451	$A_1 A_2^3$	452-55	A_2	456	$A_1 A_2^2$	457	$A_1 A_2$	458	$A_1 A_2^2$	459	$A_1 A_2^2$	460-63	A_2	464-67	$A_1 A_2$	466	$A_1 A_2^2$	467	$A_1 A_2$	
469	A_2	470	$A_1 A_2^2$	471	A_2	472	$A_1 A_2^2$	473	$A_1 A_2^2$	474-474	A_2	$A_1 A_2^2$	475	$A_1 A_2^2$	476	A_2	477	$A_1 A_2^2$	480-88	A_2	489	$A_1 A_2^3$	490-8	A_2
510	A_2	$A_1 = 30$	64-74	A_1	75	$A_1 A_2^2$	76-78	A_1	79	A_2	80-88	A_1	89-90	$A_1 A_2^2$	91	A_2	92	A_1	93	$A_1 A_2$	94-95	A_2		
96-2	A_1	103	A_2	104-6	A_1	107	A_2	108-10	A_1	111	A_2	112-14	A_1	115	A_2	116-18	A_1	119	A_2	120	A_1	121	$A_1 A_2$	
122-23	A_2	124	A_1	125		$A_1 A_2 A_1 A_2^3$	128-42	A_1	143	$A_1 A_2$	144-46	A_1	147	$A_1 A_2^2$	148-50	A_1	151	$A_1 A_2^2$	152	$A_1 A_2$	153	$A_1 A_2^2$	154	$A_1 A_2^4$
153-154		$A_1 A_2$	155	A_2																				

224-227	$A_1 A_2$	228	A_2	230	A_2	232-233	$A_1 A_2$	234-36	A_2	238	A_2	240-44	A_2	246	A_2	248	A_2			
249	$A_1 A_2 A_1 A_2^2$	250-52	A_2	254	A_2	322	$A_1 A_2$	323	$A_1 A_2^2$	326-27	A_2	330	$A_1 A_2$	331	$A_1 A_2^4$	334-35	A_2			
342-43	A_2	346-47	A_2	350-51	A_2	354-55	A_2	358	A_2	362-63	A_2	366	A_2	370-71	A_2	378-79	A_2			
393	$A_1 A_2^3$	394	$A_1 A_2^2$	395	$A_1 A_2^6$	396-99	A_2	408	$A_1 A_2^3$	409	$A_1 A_2^4$	410	$A_1 A_2^6$	411-15	A_2	424-28	A_2			
448-451	$A_1 A_2$	452-55	A_2	456-459		$A_1 A_2^2$	460-63	A_2	464-467		$A_1 A_2^2$	468	$A_1 A_2^6$	469	A_2	470	$A_1 A_2^3$			
472-473	$A_1 A_2^2$	474	$A_1 A_2^3$	475	$A_1 A_2^2$	476-80	A_2	481	$A_1 A_2 A_1 A_2^2$	482-84	A_2	486	A_2	488	A_2	489	$A_1 A_2^2$			
494	A_2	496-0	A_2	502	A_2	504-8	A_2	510	A_2	$A_1 = 38$		64	$A_1 A_2$	65	A_2	66	$A_1 A_2^2$			
72-74	$A_1 A_2$	75	A_2	76	$A_1 A_2$	77-79	A_2	80-82		$A_1 A_2$	83	$A_1 A_2^3$	84	$A_1 A_2$	85	A_2				
88-89	$A_1 A_2$	90-91	A_2	92	$A_1 A_2$	93-95	A_2	96	$A_1 A_2$	97	$A_1 A_2^2$	98	$A_1 A_2 A_1 A_2^2$	99-1	A_2	104	$A_1 A_2$			
106	$A_1 A_2$	107	A_2	108	$A_1 A_2$	109	A_2	112	$A_1 A_2$	113	$A_1 A_2^3$	114-15	A_2	116	$A_1 A_2$	120	$A_1 A_2$			
124	$A_1 A_2$	125	$A_1 A_2^5$	136		$A_1 A_2 A_1 A_2 A_1 A_2^2$		137	$A_1 A_2^2$	138		$A_1 A_2 A_1 A_2 A_1 A_2^2$	139	A_2	140					
141-43	A_2	152	$A_1 A_2^2$	153		$A_1 A_2 A_1 A_2^3 A_1 A_2^2$	154-59	A_2	168	$A_1 A_2 A_1 A_2^2 A_2$	169	$A_1 A_2^3$	170-73	A_2	184	A_2				
192-196	$A_1 A_2$	197	A_2	198	$A_1 A_2$	199	A_2	200-202		$A_1 A_2^2$	203	A_2	204	$A_1 A_2^2$	205-7	A_2				
213	A_2	214	$A_1 A_2$	215	A_2	216-218		219	A_2	220	$A_1 A_2$	221-23	A_2	224-26	$A_1 A_2$	227-29	A_2			
233	$A_1 A_2^2$	234-37	A_2	240-45	A_2	248	$A_1 A_2$	249	$A_1 A_2^3$	250-53	A_2	320	A_2	321	$A_1 A_2^4$	322-24	A_2			
329	$A_1 A_2^2$	330	$A_1 A_2$	331	$A_1 A_2^5$	332-35	A_2	336	$A_1 A_2$	337	$A_1 A_2^2$	338	$A_1 A_2^3$	339	$A_1 A_2^4$	340-43	A_2			
352	$A_1 A_2^2$	353	$A_1 A_2^2$	354-57	A_2	360	$A_1 A_2$	361	$A_1 A_2^2$	362-65	A_2	368	$A_1 A_2^2$	369	$A_1 A_2^3$	370-73	A_2			
392	$A_1 A_2^2$	393-394		$A_1 A_2^3$	395-99	A_2	408		$A_1 A_2 A_1 A_2^3 A_2$	409	$A_1 A_2^3$	410	$A_1 A_2^5$	411-15	A_2	424-29	A_2			
449-451	$A_1 A_2^2$	452-55	A_2	456	$A_1 A_2$	457-458		$A_1 A_2^2$	459	$A_1 A_2^2$	460-63	A_2	464	$A_1 A_2$	465-467	$A_1 A_2^2$	468-69	A_2		
470	$A_1 A_2^2$	471	A_2	472	$A_1 A_2$	473	$A_1 A_2^2$	474	$A_1 A_2^3$	475	$A_1 A_2^4$	476-80	A_2	481	$A_1 A_2^5$	482-85	A_2			
496-1	A_2	504	A_2	505	$A_1 A_2^4$	506-9	A_2	$A_1 = 39$		$A_1 A_2^2$	64-65		$A_1 A_2$	66-67		68	$A_1 A_2$			
71	A_2	72	$A_1 A_2^2$	73-75		$A_1 A_2$	76	$A_1 A_2^2$	77-78	A_2	$A_1 A_2$	79	A_2	80-84	$A_1 A_2$	85	A_2			
87	$A_1 A_2^3$	88-90		$A_1 A_2$	91	A_2	92-93		$A_1 A_2$	94-95	A_2	96-97	$A_1 A_2^2$	98	$A_1 A_2$	99	$A_1 A_2 A_1 A_2^2$			
100	$A_1 A_2^2$	101-2	A_2	104	$A_1 A_2^2$	105-106		$A_1 A_2$	107	A_2	108	$A_1 A_2^2$	109	$A_1 A_2$	110	A_2	112-113	$A_1 A_2$		
116	$A_1 A_2$	117	$A_1 A_2^5$	118	A_2	120-121		$A_1 A_2$	122-23	A_2	124	$A_1 A_2$	125	$A_1 A_2^2$	126	A_2	136	$A_1 A_2 A_1 A_2^2$		
137		$A_1 A_2 A_1 A_2^2$		138-139		$A_1 A_2^2$	140		$A_1 A_2 A_1 A_2^2 A_2$	141-142		$A_1 A_2^2$	143	A_2	152-153	$A_1 A_2$	154-55	A_2		
156	$A_1 A_2^2$	157-159	A_2	168		$A_1 A_2 A_1 A_2^2 A_2$	169	$A_1 A_2^2$	170-74	A_2	184	A_2	185	$A_1 A_2^4$	186-90	A_2	192-196	$A_1 A_2$		
198-206	$A_1 A_2$	207	A_2	208-218		$A_1 A_2$	219	A_2	220	$A_1 A_2$	221-23	A_2	224-227	$A_1 A_2$	228-30	A_2	231-238	$A_1 A_2$		
232-234	$A_1 A_2$	235-38	A_2	240	A_2	241	$A_1 A_2$	242-46	A_2	248-249		$A_1 A_2$	250-54	A_2	320	$A_1 A_2^2$	321	$A_1 A_2^2$		
323	$A_1 A_2^2$	324-27	A_2	328	$A_1 A_2^2$	329-330		$A_1 A_2$	331	$A_1 A_2^2$	332-35	A_2	336-338	$A_1 A_2$	339	$A_1 A_2^2$	340-43	A_2		
344-345	$A_1 A_2$	346	$A_1 A_2^3$	347-51	A_2	352	$A_1 A_2^2$	353	$A_1 A_2^2$	354-58	A_2	360	$A_1 A_2^2$	361	$A_1 A_2$	362-66	A_2			
370-74	A_2	376	$A_1 A_2$	377	$A_1 A_2^2$	378-79	A_2	380	$A_1 A_2^2$	381-82	A_2	392-394		$A_1 A_2^2$	395	$A_1 A_2^4$	396-99	A_2		
409	$A_1 A_2^3$	410	$A_1 A_2^4$	411-15	A_2	424-30	A_2	440-46	A_2	448-451		$A_1 A_2$	452-55	A_2	456-458	$A_1 A_2^2$	459	$A_1 A_2^2$		
464-466	$A_1 A_2$	467	$A_1 A_2^2$	468	$A_1 A_2^5$	469	A_2	470	$A_1 A_2^3$	471	A_2	472	$A_1 A_2$	473-474	$A_1 A_2^2$	475	$A_1 A_2^3$			
481	$A_1 A_2^3$	482-86	A_2	488	A_2	489	$A_1 A_2^5$	490-94	A_2	496-2	A_2	504	A_2	505	$A_1 A_2^3$	506-10	A_2			
70-71	$A_1 A_2$	74-75		$A_1 A_2$	78	$A_1 A_2^2$	79	A_2	82-83	$A_1 A_2$	86	$A_1 A_2^2$	87	$A_1 A_2^3$	90-91	A_2	94-95	A_2		
98-99	$A_1 A_2$	102	$A_1 A_2$	103	A_2	106	$A_1 A_2$	107	A_2	110	A_2	114-15	A_2	118-19	A_2	122-23	A_2	128	A_1	
129-131	$A_1 A_2$	132	A_1	133-135		$A_1 A_2$	136	A_1	137-139	$A_1 A_2$	140	$A_1 A_2^2$	141	A_2	142	$A_1 A_2^2$	143	A_2		
144	A_1	145-147	$A_1 A_2$	148-150		$A_1 A_2^2$	151	$A_1 A_2$	152-153	$A_1 A_2^2$	154-59	A_2	160	A_1	161-163	$A_1 A_2$	184-88	A_2		
164	A_1	165-167	$A_1 A_2$	168	A_1	169	$A_1 A_2$	170-72	A_2	174	A_2	176-78	A_2	179	$A_1 A_2$	180-82	A_2	183	$A_1 A_2$	
190	A_2	192-203		$A_1 A_2$	204	$A_1 A_2^2$	205	A_2	206	$A_1 A_2$	207	A_2	208-211	$A_1 A_2$	212	$A_1 A_2^2$	218	A_2		
219-23	A_2	224-227		$A_1 A_2$	228-29	A_2	230	$A_1 A_2$	231	A_2	232-234	$A_1 A_2$	236-27	A_2	238	A_2	245	A_2		
242-48	A_2	249	$A_1 A_2$	250-52	A_2	254	A_2	252-323		$A_1 A_2^2$	262-323	A_2	266	A_2	270-71	A_2	278-79	A_2		
342	$A_1 A_2^6$	343	A_2	346	$A_1 A_2^6$	347	A_2	350-51	A_2	354-55	A_2	358-59	A_2	362-63	A_2	366	A_2	378-79	A_2	
382	A_2	384	A_1	385-387		$A_1 A_2$	388-389		$A_1 A_2^2$	390-391		$A_1 A_2^2$	392-394		$A_1 A_2^2$	395	$A_1 A_2^2$	396-99	A_2	
400-42	$A_1 A_2^4$	401-402		$A_1 A_2^2$	403	$A_1 A_2$	404-405		$A_1 A_2^3$	406-407		$A_1 A_2^2$	408-409		$A_1 A_2^2$	410	$A_1 A_2^4$	411-18	A_2	
419	$A_1 A_2^2$	420-22	A_2	423	$A_1 A_2$	424-28	A_2	430	A_2	432-44	A_2	446	A_2	448-451	$A_1 A_2^4$	452-53	A_2	454	$A_1 A_2^3$	
456	A_2	457-459		$A_1 A_2$	459	$A_1 A_2^2$	460-63	A_2	465	$A_1 A_2^2$	467	$A_1 A_2^2$	468	$A_1 A_2^4$	469	A_2	471	$A_1 A_2^3$		
472-474	$A_1 A_2^2$	475	$A_1 A_2^3$	476-80	A_2	481	$A_1 A_2$	482-88	A_2	489	$A_1 A_2$	490-92	A_2	494	A_2	496-8	A_2	510	A_2	
474	A_1	75	$A_1 A_2$	76	A_1	77	$A_1 A_2$	78	A_1	79	A_2	80	A_1	81	$A_1 A_2^2$	82	A_1	83	$A_1 A_2^2$	
86	A_1	87	$A_1 A_2$	88	A_1	89-91		$A_1 A_2$	92	A_1	93	$A_1 A_2$	94-95	$A_1 A_2$	96-98	A_1	99	$A_1 A_2^2$	100	A_1
102	A_1	103	A_2	104	A_1	105	$A_1 A_2$	106	A_1	107	$A_1 A_2$	108	A_1	109	$A_1 A_2$	110	A_1	112	A_1	
115	$A_1 A_2$	116	A_1	117	$A_1 A_2^2$	118	A_1	119	A_2	120	A_1	121	$A_1 A_2$	124	$A_1 A_2$	125	$A_1 A_2$	126	$A_1 A_2$	
131	$A_1 A_2$	132-34	A_1	135	$A_1 A_2$	136-38	A_1	139	$A_1 A_2$	140-42	A_1	143	$A_1 A_2$	144-46	A_1	147	$A_1 A_2$	148-50	A_1	
153	$A_1 A_2$	154	$A_1 A_2^2$	155	A_2	156	A_1	157	$A_1 A_2^2$	158-59	A_2	160-62	A_1	163	$A_1 A_2$	164-66	A_1	167	$A_1 A_2$	
172-74	A_1	176-78	A_1	179	$A_1 A_2$	180-82	A_1	183	$A_1 A_2$	184	A_1	185	$A_1 A_2^2$	186-90	A_2	192	A_1	193-195	$A_1 A_2$	
197-199	$A_1 A_2$	200	A_1	201-203		$A_1 A_2$	204	A_1	205-206	$A_1 A_2$	207	A_2	208	A_1	209-211	$A_1 A_2$	211	A_2		
212	A_1 </td																			

111	A_2	112	$A_2^2 A_2$	113-114	$A_1 A_2$	115	$A_2 A_2$	116	$A_1^2 A_2$	117	$A_1 A_2^2 A_1 A_3 A_1 A_3^2$	118	A_2	120-122	$A_1 A_2$	
123	A_2	124-125	$A_1 A_2$	126	A_2	136	$A_1^2 A_2$	137-139	$A_1 A_2$	140	$A_1^2 A_2$	141-142	$A_1 A_2$	143	A_2	
152-154		$A_1 A_2$	155	A_2	156-157	$A_1 A_2$	158-59	A_2	168	$A_2^2 A_2$	169-170	$A_1 A_2$	171	A_2	172	$A_1^2 A_2$
184	$A_1 A_2$	185	$A_1 A_2^2$	186-90	A_2	192	$A_2^2 A_2$	193-195	$A_1 A_2$	196	$A_1^2 A_2$	197	A_2	198-206	$A_1 A_2$	
208-222		$A_1 A_2$	223	A_2	224	$A_2^2 A_2$	225-227	$A_1 A_2$	228	$A_2^2 A_2$	229-31	A_2	232-234	$A_1 A_2$	235	$A_1 A_2^2$
237-39	A_2	240-241	$A_1 A_2$	242-46	A_2	248-249	$A_1 A_2$	250-54	A_2	320	$A_1^2 A_2$	321	$A_2^2 A_2$	322	$A_1 A_2$	
328	$A_2^2 A_2$	329-330	$A_1 A_2$	331	$A_1 A_2^2$	332-33	A_2	336	$A_2^2 A_2$	337-338	$A_1 A_2$	339	$A_1 A_2^2$	340-43	A_2	
347	$A_1 A_2^2$	348	$A_1 A_2$	349-51	A_2	352	$A_1^2 A_2^2$	353-354	$A_2^2 A_2$	355	$A_1 A_2^3$	356-59	A_2	360	$A_2^2 A_2$	
368	$A_2^2 A_2$	369	$A_1^2 A_2^2$	370-74	A_2	376-377	$A_1 A_2$	378-79	A_2	380	$A_1 A_2$	381-82	A_2	392	$A_2^2 A_2$	
396-99	A_2	408	$A_1 A_2$	409-410	$A_1 A_2^2$	411	$A_1 A_2^3$	412-15	A_2	424	A_2	425	$A_1 A_2^2$	460-63	A_2	
448	$A_2^2 A_2$	449-451	$A_1 A_2$	452-55	A_2	456-458	$A_1 A_2$	459	$A_1 A_2^2$	464-468	$A_1 A_2$	469	A_2	470	$A_1 A_2$	
471	A_2	472-474	$A_1 A_2$	475	$A_1 A_2^2$	476-80	A_2	481	$A_1 A_2^2$	482-87	A_2	488	$A_1 A_2$	489	$A_1 A_2^2$	
506-10	A_2	501	$A_1 = 63$	64-90	A_1	91	$A_1 A_2$	92-94	A_1	95	$A_1 A_2$	96-10	A_1	111	$A_1 A_2$	
201-2	A_1	203	$A_1 A_2$	204-6	A_1	207	$A_1 A_2$	209	A_1	211	$A_1 A_2$	212-14	A_1	215	$A_1 A_2$	
228-31	A_1	237-38	A_1	239	A_2	245	A_1	247	A_2	320-34	A_1	335	A_2	336-46	A_1	
359	A_2	360-62	A_1	363	$A_1 A_2^2$	364-66	A_1	367	A_1	368-74	A_1	375	A_2	376-78	A_1	
457-58	A_1	459	$A_1 A_2$	460	A_1	461	A_1	462	A_1	463	A_1	465	A_1	467	$A_1 A_2$	
477	A_2	478	$A_1 A_2^2$	479	A_2	484	A_1	485-87	A_2	493-95	A_2	501	A_2	503	A_2	
99	A_2	100	A_1	101	A_2	102	A_1	103	A_2	104	A_1	105	$A_1 A_2$	106	A_1	
114-15	A_2	116	A_1	117-19	A_2	120	A_1	121	$A_1 A_2$	122-23	A_2	124	$A_2^2 A_1 A_2$	125	$A_1 A_2^3$	
172	A_1	173-75	A_2	178	A_2	179	$A_1 A_2^5$	180-82	A_2	183	$A_1 A_2^3$	188-91	A_2	246-47	A_2	
285		$A_1 A_2 A_1 A_2^2$	286-87	A_2	304-306	$A_1^2 A_2$	307	$A_1 A_2^2$	308	$A_1^2 A_2$	309	$A_1 A_2$	310	$A_1^2 A_2$		
314-15	A_2	316	$A_1 A_2^2$	317	$A_1 A_2$	318	$A_1 A_2^4$	319	A_2	368	$A_1^2 A_2$	369	$A_1 A_2^2$	370	A_2	
380	$A_1 A_2^4$	381	A_2	434-35	A_2	438-39	A_2	444-47	A_1	$A_1 = 79$	86-87	A_1	96-97	A_1	112-22	A_1
162-63	A_1	166-67	A_1	172-75	A_1	178-83	A_1	188-89	A_1	190-91	A_2	246	A_1	247	A_2	
319	$A_1 A_2^2$	368-70	A_1	372-74	A_1	375	A_2	376-78	A_1	379	A_2	380	A_1	381	$A_1 A_2^2$	
$A_1 = 84$		99	A_2	102-3	A_2	106-7	A_2	110-11	A_2	114-15	A_2	122-23	A_2	163	$A_1 A_2^3$	
179	A_2	181-82	A_2	183	$A_1 A_2^5$	189-91	A_1	231	A_2	271	A_2	287	A_2	291	$A_1 A_2^4$	
311	$A_1 A_2^3$	315	A_2	319	A_2	355	A_2	359	A_2	363	A_2	379	A_2	423	$A_1 A_2^5$	
$A_1 = 85$		99	A_1	102	A_1	103	A_2	106	A_1	107	A_2	110	A_1	111	A_2	
126	A_2	162-67	A_1	172-74	A_1	175	A_2	178-82	A_1	183	$A_1 A_2 A_1 A_2^3$	188-91	A_2	230	A_1	
271	$A_1 A_2^3$	286	A_1	287	A_2	290-91	A_1	294	A_1	295	$A_1 A_2 A_1$	298	A_1	302	$A_1 A_2^2$	
311	$A_1 A_2^2$	314	$A_1 A_2^7$	315	A_2	318	$A_1 A_2^3$	319	A_2	354	A_1	355	A_2	362	$A_1 A_2$	
374	A_2	378-79	A_2	382	A_2	418-22	A_1	423	$A_1 A_2^3$	428-31	A_2	434-39	A_2	444-47	A_2	
88-89		$A_1 A_2$	90-91	A_2	92	$A_1 A_2$	93	$A_1 A_2^2$	96	$A_1^2 A_2$	97	$A_1 A_2$	98	A_2	100	$A_2^2 A_2$
104-106		$A_1 A_2$	107	A_2	108	$A_1 A_2$	109-11	A_2	112	$A_1 A_2$	113	$A_1 A_2^2$	114-15	A_2	116	$A_1 A_2$
120-121		$A_1 A_2$	122-23	A_2	124	$A_1 A_2$	125	$A_1 A_2^2$	126	$A_1 A_2$	127	$A_1 A_2$	139	A_2	140	$A_1 A_2$
154-55	A_2	156	$A_1 A_2$	157-59	A_2	160	$A_1 A_2^2$	161	$A_1 A_2^3$	162	$A_1 A_2^2$	163	$A_1 A_2$	164	$A_1 A_2^5$	
170-71	A_2	172	$A_1 A_2^2$	173-82	A_2	183	$A_1 A_2^2$	184	A_2	185	$A_1 A_2^2$	186-91	A_2	200-202	$A_1 A_2$	
216-218		$A_1 A_2$	219	A_2	220	$A_1 A_2$	221	A_2	224	$A_1 A_2^2 A_1 A_2 A_1 A_2$	225-226	$A_1 A_2^2 A_1 A_2$	227-31	A_2	232-233	$A_1 A_2$
234-40	A_2	241	$A_1 A_2^5$	242-45	A_2	248	$A_1 A_2$	249	$A_1 A_2^2$	250-53	A_2	264	$A_1 A_2^3$	265	$A_1 A_2 A_1 A_2 A_1 A_2 A_1 A_2$	
269	$A_1 A_2^2$	270	$A_1 A_2$	271	$A_1 A_2^5$	280-281	A_2	282-83	A_2	284	$A_1 A_2^2$	295	$A_1 A_2^2$	296-298	A_2	
286-87	A_2	288-290	$A_1^2 A_2$	291	$A_1 A_2^2$	292	$A_1 A_2^2$	293-294	A_2	295	$A_1 A_2^2$	307	$A_1 A_2^3$	308	$A_1 A_2^2$	
301-302		$A_1 A_2$	303	$A_1 A_2^3$	304-306	$A_1^2 A_2$	317	$A_1 A_2$	318	$A_1 A_2^2 A_1 A_2^3$	319	$A_1 A_2$	320-330	$A_1 A_2$	321	$A_1 A_2^5$
314	$A_1 A_2^4$	315	A_2	316	$A_1 A_2^2 A_1 A_2^2$	324	$A_1 A_2^2$	325	A_2	326	$A_1 A_2^2 A_1 A_2^2$	327-39	A_2	328	$A_1 A_2^2$	
344-346		$A_1 A_2$	347-49	A_2	352	$A_1 A_2^2$	353	$A_1 A_2^2$	354-59	A_2	360	$A_1 A_2^2$	361	$A_1 A_2$	362-67	A_2
372	$A_2^2 A_2^7$	373	A_2	376	$A_1 A_2$	377	$A_1 A_2^2$	378-79	A_2	380	$A_1 A_2^2$	381	A_2	392	$A_1 A_2^2 A_1 A_2 A_1 A_2$	
408	$A_1 A_2$	409	$A_1 A_2^2$	410	$A_1 A_2^5$	411-12	A_2	423	$A_1 A_2^4$	424-47	A_2	456-458	$A_1 A_2$	459	$A_1 A_2^3$	
474	$A_1 A_2$	475-77	A_2	480	A_2	481	$A_1 A_2^4$	482-87	A_2	488	$A_1 A_2^2$	490-1	A_2	504	A_2	
88		$A_1^2 A_2 A_1 A_2^2$	89-90	$A_1 A_2$	91	A_2	92	$A_1^3 A_2$	93	$A_1 A_2$	94	A_2	96	A_1	97	$A_1 A_2^2 A_2$
100	A_1	101	$A_1 A_2^2$	102	A_1	103	A_2	104	$A_1^2 A_2$	105	$A_1^2 A_2$	106	$A_1 A_2$	108	$A_1^4 A_2 A_2^3 A_2 A_2^2$	
111	A_2	112	A_1	113		$A_1^3 A_2 A_1 A_2^3 A_2 A_2$		114	A_1	115	A_2	116	$A_1 A_2^2 A_2$	117	$A_1 A_2$	
122-23	A_2	124	$A_2^2 A_2$	125	$A_1 A_2$	126	A_2	136	A_1	137	$A_1 A_2^5$	138	$A_1 A_2$	139	$A_1 A_2^2 A_1 A_2$	
152	A_1	153	$A_1 A_2$	154	$A_1 A_2^2$	155	A_2	156	A_1	157	$A_1 A_2 A_1 A_2 A_1 A_2^2$	158-59	A_2	160-64	A_1	
167	$A_1 A_2$	168	$A_1 A_2^2$	169	A_2	170	$A_1 A_2^2$	171	$A_1 A_2^4$	172	$A_1 A_2^2 A_1 A_2^2$	173	A_2	176-80	A_1	
182	A_1	183	$A_1 A_2$	184	$A_1 A_2^2$	185	A_2	186-91	A_2	200	A_1	201-203	$A_1 A_2$	204	$A_1 A_2^2$	
231	A_2	232	$A_1 A_2^4$	233	$A_1 A_2$	234	$A_1 A_2^2$	235	A_2	236	$A_1 A_2^2 A_1 A_2^2$	237-39	A_2	240	A_1	
250-54	A_2	264	A_1	265	$A_1 A_2^2$	266	A_1	267	$A_1 A_2^2$	268	$A_1 A_2^2$	269	$A_1 A_2^2$	270	$A_1 A$	

263	A ₁ A ₂	264-66	A ₁	267	A ₂	268	A ₁	269	A ₁ A ₂	270	A ₁	271	A ₁ A ₂ ²	274-76	A ₁	277	A ₁ A ₂	278	A ₁	279	A ₁ A ₂	280	A ₁						
281	A ₁ A ₂	282-83	A ₂	284	A ₁	285	A ₁ A ₂	286-87	A ₂	288-92	A ₁	293	A ₁ A ₂	294	A ₁	295	A ₁ A ₂	296	A ₁	297	A ₁ A ₂	298	A ₁						
299	A ₁ A ₂ ⁴	300	A ₁ A ₂	301	A ₁ A ₂	302	A ₁ A ₂ ² A ₂ ²	303	A ₁ A ₂ ²	304-6	A ₁	307	A ₁ A ₂ ³	308	A ₁	309	A ₁ A ₂	310	A ₁ A ₂ ²	311	A ₁ A ₂ ²	312	A ₁ A ₂						
312	A ₁	313	A ₁ A ₂	314	A ₂ ^{2A₂⁶}	315	A ₂	316	A ₁ A ₂	317	A ₁ A ₂	318	A ₂ ² A ₂ ⁴	319	A ₂	320-23	A ₁	324-27	A ₂	328	A ₁	329	A ₁ A ₂						
330	A ₁	331	A ₁ A ₂ ⁴	332-35	A ₂	336-38	A ₁	339	A ₁ A ₂ ²	340-43	A ₂	344	A ₁	345	A ₁ A ₂	346-51	A ₂	352	A ₁	353	A ₂ ² A ₂ ²	354	A ₁						
356-58	A ₂	360	A ₁	361	A ₁ A ₂ ²	362	A ₂	364-66	A ₂	368	A ₁	369	A ₂ ² A ₂ ³	370	A ₂	372	A ₂ ² A ₂ ²	373-74	A ₂	376	A ₁ A ₂ ²	377	A ₁ A ₂ ²						
378	A ₂	380	A ₂ ² A ₂ ⁴	381-82	A ₂	384-88	A ₁	389	A ₁ A ₂ ²	390	A ₁	391	A ₁ A ₂ ²	392-94	A ₁	395	A ₂	396	A ₁	397-99	A ₂	400-2	A ₁						
403	A ₁ A ₂	404	A ₁	405	A ₁ A ₂ ²	406	A ₁	407	A ₁ A ₂	408	A ₁	409	A ₁ A ₂ ²	410	A ₁ A ₂ ⁶	411-22	A ₂	423	A ₁ A ₂ ²	424-47	A ₂	448-50	A ₁						
451	A ₁ A ₂	452-55	A ₂	456	A ₁	457	A ₂ ² A ₂	458	A ₁ A ₂	459	A ₁ A ₂ ²	460-63	A ₂	464	A ₁	465	A ₁ A ₂	466-467	A ₂	468	A ₁ A ₂ ²	469-482	A ₂						
469	A ₂	470	A ₁ A ₂ ³	471	A ₂	472	A ₁	473	A ₁ A ₂	474	A ₁ A ₂ ²	475-82	A ₂	484-86	A ₂	488	A ₂	489	A ₂ ² A ₂ ³	490	A ₂	492-94	A ₂						
496-98	A ₂	500-2	A ₂	504-6	A ₂	508-10	A ₂	A ₁ =102	104-106	A ₁ A ₂	107	A ₂	108	A ₂	109	A ₂	109	A ₂	109	A ₂	109	A ₂							
112	A ₁	113	A ₁ A ₂	114-15	A ₂	116	A ₁	117	A ₂	120-121	A ₁ A ₂	122-23	A ₂	124	A ₁ A ₂	125	A ₁ A ₂ ³	128-30	A ₁	131	A ₂ ² A ₂	132	A ₁						
132-34	A ₁	135	A ₂ ² A ₂	136-38	A ₁	139	A ₂	140	A ₁	141	A ₂	142	A ₁	143	A ₂	144-46	A ₁	147	A ₂ ² A ₂	148-50	A ₁	151	A ₁ A ₂ ²						
152	A ₁ A ₂ ²	153	A ₁ A ₂ ² A ₁ A ₂ ³	154-59	A ₂	160-62	A ₁	163	A ₂ ² A ₂ ²	164-66	A ₁	167	A ₁ A ₂ ²	168	A ₁	169	A ₁ A ₂ ²	170-71	A ₂	172	A ₁	173-84	A ₂						
200-202	A ₁ A ₂	203	A ₂	204	A ₁	204	A ₁ A ₂	205-7	A ₂	208-212	A ₁ A ₂	213	A ₂	214	A ₁ A ₂	215	A ₁ A ₂ ²	216-218	A ₁ A ₂	219	A ₁ A ₂	220-202	A ₁ A ₂						
219	A ₂	220	A ₁	221-23	A ₂	224-30	A ₁	232	A ₁	233	A ₁ A ₂	234	A ₁	235	A ₂	236	A ₁	237-38	A ₂	240-42	A ₁	243	A ₂	244	A ₁				
314-15	A ₂	316	A ₁ A ₂ ²	317	A ₁ A ₂ ² A ₁ A ₂ ²	318-19	A ₂	320	A ₁	321	A ₁ A ₂ ²	322	A ₁	323	A ₁ A ₂	324-27	A ₂	328-330	A ₁ A ₂	329	A ₁ A ₂	331	A ₁ A ₂						
331	A ₁ A ₂ ⁴	332-35	A ₂	336	A ₁	337-38	A ₁ A ₂	339	A ₁ A ₂ ²	340-43	A ₂	344-345	A ₁ A ₂	346	A ₁ A ₂ ² A ₁ A ₂ ²	347-51	A ₂												
352	A ₂ ^{2A₂}	353	A ₁ A ₂	354-57	A ₂	360	A ₂ ^{2A₂}	361	A ₁ A ₂	362-65	A ₂	368	A ₂ ² A ₂ ²	369	A ₁ A ₂ ³	370-73	A ₂	376	A ₁ A ₂	377	A ₁ A ₂ ²	378-79	A ₂						
380	A ₁ A ₂ ⁴	381	A ₂	384-86	A ₁	387	A ₁ A ₂	388	A ₁	389	A ₁ A ₂ ²	390-391	A ₁ A ₂	392	A ₁	393-394	A ₁ A ₂ ²	395-99	A ₂	400	A ₁ A ₂ ³	409	A ₁ A ₂ ³						
400	A ₁	401	A ₁ A ₂	402	A ₁ A ₂ ³	403	A ₁ A ₂	404	A ₁ A ₂ ²	405-406	A ₁ A ₂ ²	407	A ₁ A ₂ ²	408	A ₁ A ₂ ² A ₁ A ₂ ³	408-458	A ₁ A ₂	415	A ₁ A ₂ ²	416-63	A ₂	420	A ₁ A ₂ ²						
410	A ₁ A ₂ ⁴	411-22	A ₂	423	A ₁ A ₂ ²	424-47	A ₂	448-451	A ₁ A ₂ ²	452-55	A ₂	456-458	A ₁ A ₂ ²	459	A ₁ A ₂ ²	460-63	A ₂	474	A ₁ A ₂ ²	475-476	A ₂	476-80	A ₂						
464-466	A ₁ A ₂	467	A ₁ A ₂ ²	468	A ₁ A ₂ ⁴	469	A ₂	470	A ₁ A ₂ ²	471	A ₂	472	A ₁ A ₂	473	A ₁ A ₂ ²	474	A ₁ A ₂ ²	475	A ₁ A ₂ ²	476-80	A ₂								
481	A ₁ A ₂ ⁴	482-85	A ₂	488	A ₂	489	A ₁ A ₂ ²	490-93	A ₂	496-1	A ₂	504	A ₂	505	A ₁ A ₂ ⁴	506-9	A ₂	A ₁ =103	104-6	A ₁	107	A ₁ A ₂	108	A ₁ A ₂					
108	A ₁	109	A ₁	110	A ₁	112-18	A ₂	120	A ₁	121	A ₁	122	A ₁	123	A ₂	124	A ₁	125	A ₁ A ₂	126	A ₂	128-54	A ₁						
155	A ₂	156-58	A ₁	159	A ₂	160-74	A ₁	175	A ₂	176-85	A ₁	186-87	A ₁	188	A ₁	189-91	A ₂	192-98	A ₁	199	A ₁ A ₂	200	A ₁	201	A ₂	204	A ₁		
201	A ₁ A ₂	202	A ₁	203	A ₁ A ₂	204	A ₁	205	A ₁ A ₂	206	A ₁	207	A ₂	208-14	A ₁	215	A ₁ A ₂	216	A ₁	217	A ₁ A ₂	218	A ₁	219	A ₂				
219	A ₂	220	A ₁	221-23	A ₂	224-30	A ₁	232	A ₁	233	A ₁ A ₂	234	A ₁	235	A ₂	236	A ₁	237-38	A ₂	240-42	A ₁	243	A ₂	244	A ₁				
244	A ₁	245-46	A ₂	248	A ₁	249	A ₁ A ₂	250-54	A ₂	258-71	A ₁	274-82	A ₁	283	A ₂	284-86	A ₁	287	A ₂	288-2	A ₁	303	A ₁ A ₂	303	A ₁ A ₂				
304-14	A ₁	315	A ₂	316	A ₁	317	A ₂ ² A ₂	318	A ₁	319	A ₂	320-24	A ₁	325	A ₂	326	A ₁	327	A ₂	328-30	A ₁	331	A ₁ A ₂	332	A ₁ A ₂				
332	A ₁	333	A ₂	334	A ₁	335	A ₂	336-40	A ₁	341	A ₂	342	A ₁	343	A ₂	344	A ₁	345	A ₁ A ₂	346	A ₁	347	A ₂	348	A ₁				
348	A ₁	349-51	A ₂	352-56	A ₁	357	A ₂	358	A ₁	360	A ₁	361	A ₂ ² A ₂	362	A ₁	363	A ₂	364	A ₁	365-66	A ₂	368-70	A ₁	369-71	A ₂	370-71	A ₁		
371	A ₂	372	A ₁	373-74	A ₂	376	A ₁	377	A ₁ A ₂	378-79	A ₂	380	A ₂ ² A ₂	381	A ₁ A ₂	382	A ₂	384-98	A ₁	393	A ₂	399	A ₂	400-6	A ₁				
411	A ₂	412	A ₁	413-15	A ₂	416-26	A ₁	427	A ₂	428	A ₁	429-47	A ₂	430-451	A ₁	440-451	A ₂	445-52	A ₁	451	A ₁ A ₂	452-453	A ₂	454	A ₁	455	A ₂		
455	A ₂	456	A ₁	457	A ₁ A ₂	458	A ₁	460	A ₁ A ₂	467	A ₁ A ₂ ²	468	A ₁ A ₂ ²	469	A ₂	470	A ₁ A ₂ ²	471	A ₁ A ₂ ²	474	A ₁ A ₂ ²	475-79	A ₂	482-87	A ₂				
490	A ₂	492	A ₂	494	A ₂	498-3	A ₂	506	A ₂	508	A ₂	510	A ₂	510	A ₂	A ₁ =107	108-10	A ₁	112-18	A ₁	119	A ₂	120-22	A ₁	120	A ₂			
114-15	A ₂	118-19	A ₂	122-23	A ₂	129-131	A ₁	A ₁ A ₂	133-135	A ₁ A ₂	137-138	A ₁ A ₂	139	A ₂	140-42	A ₁	143	A ₁ A ₂	152-54	A ₁	155	A ₂	156-58	A ₁	157-59	A ₂	158-60	A ₁	
143	A ₂	144-147	A ₁	149	A ₁ A ₂	150-151	A ₁ A ₂	153	A ₁ A ₂ ³	154-55	A ₂	157-59	A ₂	161	A ₁ A ₂	162	A ₁	163	A ₁ A ₂	164-66	A ₁	167	A ₁ A ₂	168-70	A ₁	171	A ₂	172-74	A ₁
165-167	A ₁ A ₂	169	A ₁ A ₂	170-71	A ₂	173-75	A ₂	177	A ₂	179	A ₁ A ₂ ²	181-82	A ₂	183	A ₁ A ₂ ³	185-87	A ₂	189-91	A ₂	193	A ₁ A ₂	194-95	A ₂	195	A ₁ A ₂				
195	A ₁ A ₂	197	A ₂																										

347	A_2	350-51	A_2	354	A_1	355	$A_1 A_2$	358-59	A_2	362	A_1	363	A_2	366	A_2	370-71	A_2	374-75	A_2	378-79	A_2	382	A_2
384-86	A_1	387	$A_1 A_2$	388	A_1	389-391	$A_1 A_2$	392-94	A_1	395	$A_1 A_2$	396	A_1	397	$A_1 A_2$	398-399	$A_1 A_2$	400-2	A_1				
403	$A_1 A_2$	404	A_1	405	$A_1 A_2^2$	406-407	$A_1 A_2$	408	A_1	409	$A_1 A_2^3$	410	$A_1 A_2$	411-15	A_2	416-18	A_1	419	$A_1 A_2$	420	A_1		
421	$A_1 A_2^2$	422-423	$A_1 A_2$	424-25	A_1	426-38	A_2	439	$A_1 A_2^2$	440-47	A_2	448	A_1	449-451	$A_1 A_2$			452	$A_1 A_2^2$	453	A_2		
454-455	$A_1 A_2$	456	A_1	457-459	$A_1 A_2$	460-61	A_2	462	$A_1 A_2^2$	463	A_2	464	A_1	465-467	$A_1 A_2$			468	$A_1 A_2^2$	469	$A_1 A_2^2$		
469	$A_1 A_2^3$	470-471	$A_1 A_2^2$	472	A_1	473-475	$A_1 A_2^2$	476-79	A_2	480	A_1	481	$A_1 A_2$	482	A_2	483	$A_1 A_2$	484-87	A_2				
488	A_1	489	$A_1 A_2$	490-92	A_2	494	A_2	496-8	A_2	510	A_2	A₁ = 110		112-14	A_1	115	A_2	116-18	A_1	119	A_2	120	A_1
121	$A_1 A_2$	122-23	A_2	124	A_1	125	$A_1 A_2^2$	128-34	A_1	135	$A_1 A_2$	136-38	A_1	139	$A_1 A_2^2$	140-42	A_1	143	A_2	144-46	A_1	147	$A_1 A_2$
148-50	A_1	151	$A_1 A_2$	152-53	A_1	154	$A_1 A_2^2$	155	A_1	156	A_1	157-59	A_2	160-66	A_1	167	$A_1 A_2$	168-70	A_1	171	A_2	172-74	A_1
175	A_2	176-78	A_1	179	$A_1 A_2^2$	180-82	A_1	183	$A_1 A_2^2$	184	A_1	185	$A_1 A_2^3$	186-91	A_2	192-94	A_1	195	$A_1 A_2$	196	A_1	197	$A_1 A_2^2$
198-199	$A_1 A_2$	200	A_1	201-203	$A_1 A_2$	204	A_1	205	A_2	206	$A_1 A_2$	207	A_2	208	A_1			209-211	$A_1 A_2$				
212	A_1	213-218	$A_1 A_2$	219	A_2	220	$A_1 A_2$	221-23	A_2	224-25	A_1	226-227	$A_1 A_2^2$	231	$A_1 A_2$	241	$A_1 A_2$	242-47	A_2	248-249	$A_1 A_2^2$		
231	A_2	232	A_1	233-234	$A_1 A_2$	235	A_2	236	A_1	237	A_2	240	A_1	241	$A_1 A_2$	242-47	A_2	248-249	$A_1 A_2^2$				
250-53	A_2	258-62	A_1	263	$A_1 A_2^2$	264-70	A_1	271	$A_1 A_2$	274-75	A_1	279	$A_1 A_2$	280-82	A_1	283	A_2	284-86	A_1	287	A_2	288-94	A_1
295	$A_1 A_2$	296-2	A_1	303	$A_1 A_2^2$	304-6	A_1	307	$A_1 A_2$	308-10	A_1	311	$A_1 A_2^2$	312-13	A_1	314	$A_1 A_2^4$	315	A_2	316	A_1	317	$A_1 A_2$
318	$A_1 A_2^2$	319	A_2	320-22	A_1	323	$A_1 A_2^2$	324	A_1	325	A_1	326	A_1	328-30	A_1	331	$A_1 A_2^2$	332	A_1	333-35	A_2		
336-38	A_1	339	$A_1 A_2^2$	340	A_1	341	A_2	342	$A_1 A_2$	343	A_2	344	A_1	345-346	$A_1 A_2$	347	A_2	348	A_1	349-51	A_2		
352-54	A_1	355	A_2	356	A_1	357-59	A_2	360	A_1	361	$A_1 A_2$	362	A_1	363	A_2	365	A_1	368	A_1	369	$A_1 A_2^2$		
370-71	A_2	372	$A_1 A_2^4$	373-75	A_2	376	A_1	377	$A_1 A_2^2$	378-79	A_2	380	$A_1 A_2^2$	381	A_2	384-86	A_1	387	$A_1 A_2$	388-89	A_1		
390-391	$A_1 A_2$	392-94	A_1	395	$A_1 A_2$	396	A_1	397	A_2	398	$A_1 A_2$	399	A_2	400-2	A_1	403	$A_1 A_2$	404	A_1	405	$A_1 A_2^2$		
406-407	$A_1 A_2$	408	A_1	409-410	$A_1 A_2^2$	411-15	A_2	416-18	A_1	419	$A_1 A_2$	420	A_1	421	$A_1 A_2^2$	422-423	$A_1 A_2$						
424-25	A_1	426-38	A_2	439	$A_1 A_2^3$	440-47	A_2	448	A_1	449-451	$A_1 A_2$	452	$A_1 A_2^2$	453	A_2	454	$A_1 A_2$	455	A_2	456	A_1		
457-459	$A_1 A_2$	460-63	A_2	464	A_1	465-467	$A_1 A_2$	468	$A_1 A_2^2$	471	A_2	472	A_1	473	$A_1 A_2^2$	474	$A_1 A_2$	475	A_2	477-79	A_2	482	A_2
474-475	$A_1 A_2^2$	476-79	A_2	480	A_1	481	$A_1 A_2^2$	482-87	A_2	488	A_1	489	$A_1 A_2^2$	490-93	A_2	496-4	A_2	505	$A_1 A_2^3$	506-9	A_2		
A₁ = 111		112-22	A_1	123	A_2	124-26	A_1	128-58	A_1	159	A_2	160-86	A_1	187	A_2	188-90	A_1	191	A_2	192-6	A_1	207	$A_1 A_2$
208-14	A_1	215	$A_1 A_2$	216-18	A_1	219	$A_1 A_2$	220-22	A_1	223	A_2	224-30	A_1	231	$A_1 A_2$	232-34	A_1	235	$A_1 A_2$	236-38	A_1	240-42	$A_1 A_2$
243	$A_1 A_2$	244-46	A_1	247	$A_1 A_2$	248-50	A_1	251	A_2	252	A_1	253-54	A_2	258-71	A_1	274-34	A_1	275	A_2	233-42	A_1	343	$A_1 A_2$
344-46	A_1	347	$A_1 A_2$	348-50	A_1	351	A_2	352-58	A_1	359	A_1	360-66	A_1	368-74	A_1	375	A_2	376-78	A_1	379	A_2	380-81	A_1
382	A_2	384-14	A_1	415	A_2	416-30	A_1	431	A_2	432-38	A_1	439	$A_1 A_2$	440-41	A_1	442	A_2	444-47	A_2	448-54	A_1	455	$A_1 A_2$
456-58	A_1	459	$A_1 A_2$	460	A_1	461	A_2	462	A_1	463	A_2	464-66	A_1	467	$A_1 A_2$	468-70	A_1	471	$A_1 A_2$	472-74	A_1	475	$A_1 A_2$
476	A_1	477	A_2	478	$A_1 A_2^2$	480	$A_1 A_2$	482-80	A_1	483	$A_1 A_2$	484	A_1	485	A_2	486	A_1	487	A_2	488-90	A_1	491	$A_1 A_2$
492	A_1	493	A_2	496-97	A_1	498-0	A_2	502-3	A_2	504	A_1	505	$A_1 A_2$	506-9	A_2	A₁ = 113		114	A_2	118	A_2	122	A_2
130	$A_1 A_2^4$	131	$A_1 A_2^3$	132	$A_1 A_2^2$	133	$A_1 A_2$	134	$A_1 A_2^2$	135	$A_1 A_2^2$	138	$A_1 A_2^3$	139	A_2	140	$A_1 A_2^5$	141	A_2	142	$A_1 A_2^2$	143	A_2
146-147	$A_1 A_2^2$	148-149	$A_1 A_2$	150	$A_1 A_2^2$	151	$A_1 A_2^2$	167	$A_1 A_2^2$	171-74	A_2	178	A_2	179	A_2	180	$A_1 A_2^2$	184	A_2	185	$A_1 A_2^2$	186	$A_1 A_2^2$
165	$A_1 A_2 A_1 A_2 A_1 A_2 A_1 A_2^2$	166	$A_1 A_2^2$	167	$A_1 A_2^2$	195	$A_1 A_2^2$	196	$A_1 A_2^2$	197	A_2	198	$A_1 A_2^2$	199	A_2	202	$A_1 A_2$	203	A_2	204	$A_1 A_2^2$	205	$A_1 A_2^2$
187-88	A_2	190-91	A_2	194	$A_1 A_2^2$	195	$A_1 A_2^2$	213	A_2	214	$A_1 A_2$	215	$A_1 A_2^2$	218	$A_1 A_2$	220	A_2	223	A_2	227	A_2	229-30	A_2
212	$A_1 A_2 A_1 A_2 A_1 A_2 A_1 A_2^2$	213	A_2	214	$A_1 A_2$	215	$A_1 A_2^2$	218	$A_1 A_2$	220	A_2	223	$A_1 A_2$	226	$A_1 A_2$	227	A_2	229-211	$A_1 A_2$				
234	A_2	236	A_2	242	A_2	244	A_2	250	A_2	258	$A_1 A_2^2$	259	$A_1 A_2^2$	262	$A_1 A_2$	266	$A_1 A_2^2$	270	$A_1 A_2$	271	$A_1 A_2^2$	271	$A_1 A_2^2$
274	$A_1 A_2^3$	275	$A_1 A_2^2$	278	$A_1 A_2^2$	279	$A_1 A_2^2$	286	A_2	290	$A_1 A_2^2$	291	$A_1 A_2^2$	294-295	A_2								
302	$A_1 A_2$	303	$A_1 A_2^2$	306	$A_1 A_2$	307	$A_1 A_2^2$	310	$A_1 A_2^2$	311	$A_1 A_2^2$	314	$A_1 A_2^2$	316	$A_1 A_2^2$	318	$A_1 A_2^2$	319	$A_1 A_2^2$	320-24	A_1	326	A_1
328-30	A_1	331	$A_1 A_2^2$	332	A_1	334-35	A_2	336-40	A_1	342	A_1	344	A_1	345	$A_1 A_2^2$	346	$A_1 A_2$	348	A_1	352-56	A_1	358-59	A_2
360	A_1	361	$A_1 A_2^2$	362	A_1	364	A_1	367	A_2	368-70	A_1	372	A_1	376	$A_1 A_2^2$	380	$A_1 A$						

187	A ₂	190-91	A ₂	201	A ₁	203	A ₁ A ₂	205	A ₁	207	A ₂	210-11	A ₁	214-15	A ₁	217-219	A ₁ A ₂	221-222	A ₁ A ₂	251	A ₂	253	A ₁ A ₂ ⁵									
223	A ₂	233	A ₁	235	A ₁ A ₂	237	A ₁ A ₂ A ₁ A ₂	242	A ₁	243	A ₁ A ₂	246	A ₁	249-250	A ₁ A ₂	345	A ₁	347	A ₁ A ₂													
292-95	A ₁	300-3	A ₁	308-11	A ₁	316-18	A ₁	319	A ₁ A ₂ ²	329	A ₁	331	A ₁	333	A ₁	335	A ₂	345	A ₁	347	A ₁ A ₂											
349	A ₁	A ₂ ^{2A₁A₂}	356-58	A ₁	359	A ₂	361	A ₁	363	A ₁ A ₂	364	A ₁	365	A ₁ A ₂ ²	366	A ₁	367	A ₂	372	A ₁	373	A ₂ ² A ₁ ³										
374	A ₁	377	A ₁ ^{2A₂}	379	A ₂	380	A ₁	381	A ₁ A ₂ ²	382	A ₂	402-3	A ₁	406-7	A ₁	410	A ₁	411	A ₁ A ₂	414	A ₁	415	A ₂									
420-23	A ₁	428-30	A ₁	431	A ₂	434-39	A ₁	442	A ₂	444-47	A ₂	457	A ₁	459	A ₁ A ₂	461	A ₂	463	A ₂	466	A ₁	467	A ₁ A ₂									
470	A ₁	471	A ₁ A ₂	473-475	A ₁ A ₂	477-79	A ₂	484	A ₁	485-87	A ₂	489	A ₁ A ₂ ²	491	A ₁ A ₂	492-93	A ₂	498-0	A ₂	502	A ₂											
505	A ₁ A ₂	506-9	A ₂	A ₁ = 121		122	A ₂	126	A ₂	130-131	A ₂ ^{2A₂}	132	A ₁ A ₂ ²	133	A ₁ A ₂ ²	134	A ₁ A ₂ ²	135	A ₁ A ₂	138	A ₂ ² A ₁											
139	A ₁ A ₂	140	A ₁ ^{3A₂}	141	A ₁ A ₂ ²	142-143	A ₁ A ₂	146-147	A ₁ A ₂	148	A ₁ A ₂ ²	149	A ₁ A ₂ A ₁ A ₂ ²	150-151	A ₁ A ₂																	
154	A ₁ A ₂	155	A ₂	156	A ₁ A ₂ ²	157	A ₂	158	A ₁ A ₂	159	A ₂	162	A ₁ A ₂ ²	163	A ₁ A ₂	164	A ₁ A ₂ ²	165	A ₁ A ₂ ² A ₁ A ₂ ²													
166-167	A ₁ A ₂			170-171	A ₁ A ₂	172	A ₁ A ₂ ²	173	A ₁ A ₂ ²	174	A ₁ A ₂	175	A ₂	178-179	A ₁ A ₂	180-181	A ₁ A ₂															
182-183	A ₁ A ₂			187-91	A ₂	194	A ₁ A ₂ ²	195	A ₁ A ₂	196	A ₁ A ₂ ²	197	A ₁ A ₂ ²	198-199	A ₂																	
205	A ₂	206	A ₁ A ₂	207	A ₂	210-211	A ₁ A ₂	212	A ₁ A ₂ ² A ₁ A ₂ ²	213	A ₁ A ₂ ²	214-215	A ₁ A ₂	218	A ₁ A ₂	219	A ₂	204	A ₁ A ₂ ²													
220	A ₁ A ₂ ²	221-23	A ₂	226-227	A ₁ A ₂	228	A ₁ A ₂ ²	229	A ₂	230	A ₁ A ₂	231	A ₂	234	A ₁ A ₂	235-37	A ₂	242	A ₁ A ₂	243-44	A ₂											
246-47	A ₂	250	A ₂	252	A ₂	258	A ₁ A ₂ ²	259	A ₁ A ₂ ²	262	A ₁ A ₂ ²	263	A ₁ A ₂	266	A ₁ A ₂ ²	267	A ₁ A ₂	270-271	A ₁ A ₂	274	A ₁ A ₂ ²											
275	A ₁ A ₂			278-279	A ₁ A ₂	282	A ₁ A ₂	286	A ₁ A ₂	287	A ₂	290	A ₁ A ₂ ²	291	A ₁ A ₂	294-295	A ₁ A ₂	298	A ₂ ² A ₂	299	A ₁ A ₂											
302-303	A ₁ A ₂			306-307	A ₁ A ₂	310-311	A ₁ A ₂	314	A ₁ A ₂	315	A ₂	318	A ₁ A ₂	319	A ₁ A ₂ ²	322	A ₁ A ₂ ²	323	A ₁ A ₂													
326	A ₁ A ₂	327	A ₂	330	A ₁ A ₂ ²	331	A ₁ A ₂	334-35	A ₂	338-339	A ₁ A ₂	342	A ₁ A ₂	343	A ₂	346	A ₁ A ₂	347	A ₂	354	A ₁ A ₂ ²											
355	A ₂	358-59	A ₂	362-63	A ₂	366-67	A ₂	370	A ₂	374-75	A ₂	378	A ₂	382	A ₂	386	A ₁ A ₂ ²	387	A ₁ A ₂	388	A ₁ A ₂ ²	389	A ₁ A ₂									
390-391	A ₁ A ₂			394-395	A ₁ A ₂	396	A ₁ A ₂ ²	397	A ₂	398	A ₁ A ₂	399	A ₂	402-403	A ₁ A ₂	410-412	A ₂	418-419	A ₁ A ₂	420	A ₂	421	A ₁ A ₂ ²									
405	A ₁ A ₂ ² A ₁ A ₂ A ₁ A ₂ ³			406-407	A ₁ A ₂	410	A ₁ A ₂	411-12	A ₂	414-15	A ₂	418-419	A ₁ A ₂	420	A ₂	421	A ₁ A ₂ ²			452-53	A ₂	454	A ₁ A ₂									
422-423	A ₁ A ₂	426	A ₂	428-31	A ₂	434-38	A ₂	439	A ₁ A ₂ ²	442	A ₂	444-47	A ₂	450-451	A ₁ A ₂	452-53	A ₂	454	A ₁ A ₂													
455	A ₂	458-459	A ₁ A ₂	460-63	A ₂	466-467	A ₁ A ₂	468	A ₁ A ₂ ²	469	A ₂	470	A ₁ A ₂	471	A ₁ A ₂ ²	474	A ₁ A ₂	475	A ₁ A ₂ ²													
476-79	A ₂	482-83	A ₂	485-87	A ₂	490-93	A ₂	498-0	A ₂	502-3	A ₂	506	A ₂	508	A ₂	A ₁ = 123		124-26	A ₁	128-58	A ₁	159	A ₁ A ₂									
160-86	A ₁	187	A ₂	188-90	A ₁	191	A ₂	192-18	A ₁	219	A ₁ A ₂	220-22	A ₁	223	A ₂	224-38	A ₁	240-46	A ₁	247	A ₁ A ₂ ²	248-50	A ₁									
252	A ₁	253	A ₂	258-71	A ₁	274-18	A ₁	319	A ₁ A ₂	320-34	A ₁	335	A ₂	336-50	A ₁	352-66	A ₁	367	A ₂	368-74	A ₁	375	A ₂									
376-78	A ₁	380-81	A ₁	382	A ₂	384-14	A ₁	415	A ₂	416-30	A ₁	431	A ₂	432-38	A ₁	439	A ₁ A ₂	440-41	A ₁	442	A ₂	444	A ₁									
445-47	A ₂	448-62	A ₁	463	A ₂	464-70	A ₁	471	A ₁ A ₂	472-74	A ₁	475	A ₁ A ₂	476-79	A ₁	477-79	A ₂	480-84	A ₁	485	A ₂	486-81	A ₁									
487	A ₂	488-83	A ₂	A ₁ = 125		492	A ₁	493	A ₂	496-9	A ₂	502-8	A ₂	A ₁ = 126		146-47	A ₁	150-51	A ₁	154	A ₁	155	A ₂ ^{2A₁}	158	A ₁							
136	A ₁	137	A ₁ ^{5A₂}	138	A ₁ ^{3A₂}	139	A ₁ ^{2A₂}	140	A ₁	141-142	A ₁ A ₂ ²	143	A ₁ A ₂	144	A ₁	145-146	A ₁ A ₂ ²	147	A ₁ A ₂ ²													
148	A ₁ A ₂ ²	149-150	A ₂	151	A ₁ ^{5A₂}	152	A ₁ A ₂	153-155	A ₁ A ₂	156	A ₁ A ₂ ²	157	A ₂	158	A ₁ A ₂	159	A ₂	161	A ₁ A ₂ ³													
159	A ₂	160	A ₁	161	A ₁ ^{5A₂}	162	A ₁ ^{3A₂}	163	A ₁ ^{2A₂}	164	A ₁	165	A ₁ A ₂ ² A ₁ A ₂ ²	166	A ₁ A ₂ ²	167	A ₂ ² A ₁	168	A ₁	169	A ₁ A ₂ ⁵											
170	A ₁ A ₂ ²	171	A ₁ A ₂	172	A ₁	173-174	A ₁ A ₂ ²	175	A ₁ A ₂	176	A ₁	177	A ₁ A ₂ ²	178	A ₁ A ₂ ²	179	A ₁ A ₂	180	A ₁ A ₂ ²	181	A ₁ A ₂											
181	A ₁ A ₂ ²	182	A ₁ ^{2A₂}	183	A ₁ A ₂	184	A ₁	185	A ₁ A ₂	186-91	A ₂	192	A ₁	193	A ₁ A ₂ ²	194	A ₁ A ₂ ²	195	A ₁ A ₂	196	A ₁ A ₂ ²											
197	A ₁ A ₂	198	A ₁ ^{2A₂}	199	A ₁ A ₂	200	A ₁	201-203	A ₁ A ₂	220	A ₁ A ₂ ² A ₁ A ₂ ²	221	A ₂	222	A ₁ A ₂	223	A ₂	224	A ₁	225	A ₁ A ₂											
226	A ₁ A ₂ ²	227	A ₁ A ₂	228	A ₁ A ₂ ²	229																										

406	$A_1^3 A_2^2$	407	$A_1 A_2$	408	$A_1^4 A_2^2$	409–410	$A_1^2 A_2^2$	412	A_2	416–18	A_2	419	$A_1^2 A_2$	420	A_2	421	$A_1^2 A_2^5$	422	$A_1^3 A_2$	423	$A_1 A_2$			
424	A_2	425	$A_1^2 A_2^5$	426	A_2	428	A_2	431	A_2	439	A_2	442	A_2	444	A_2	446	A_2	448	$A_1^2 A_2$	449	$A_1^3 A_2$			
451–452	$A_2^2 A_2$	454	$A_1^2 A_2$	456	$A_1^3 A_2$	457	$A_1 A_2$	458	$A_1^2 A_2$	459	$A_1 A_2$	460–61	A_2	463	A_2	464	$A_1^2 A_2$	465	$A_1^3 A_2$	450	$A_1^3 A_2$			
466	$A_1^3 A_2 A_1^2 A_2 A_2^2$	467	$A_1^2 A_2 A_1 A_2 A_1 A_2$	468	$A_1^2 A_2 A_1 A_2 A_1 A_2$	469	$A_1^2 A_2 A_1 A_2 A_1 A_2$	470	$A_1^2 A_2$	471	$A_1 A_2$	472	$A_1^2 A_2$	473	$A_1 A_2$	474	$A_1^2 A_2^2$	474	$A_1^2 A_2^2$	474	$A_1^2 A_2^2$			
475	$A_1 A_2^2$	478	A_2	480	$A_1^3 A_2$	481	$A_1 A_2$	482	A_2	485–87	A_2	488	$A_1^2 A_2$	489	$A_1 A_2$	491–93	A_2	498–0	A_2	502–3	A_2	505–8	A_2	
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$A_1 = 287$																								
363	$A_1 A_2 A_1 A_2$	364–66	A_1	367	A_2	372–74	A_1	375	A_2	377	$A_1 A_2$	379	A_2	380	A_1	381	$A_1 A_2$	382	A_2	402–3	A_1			
406–7	A_1	410	A_1	411	$A_1 A_2$	414	$A_1^3 A_2$	420–23	A_1	428–30	A_1	431	A_2	434–39	A_1	442	A_2	444–46	A_2	457	A_1	459	$A_1 A_2$	
461	A_2	463	A_2	466	A_1	467	$A_1^3 A_2$	470	A_1	471	$A_1 A_2$	473	$A_1 A_2$	474	$A_1^3 A_2$	475	$A_1 A_2$	477–78	A_2	484	A_1	485–87	A_2	
489	$A_1 A_2$	491	$A_1 A_2^2$	492–93	A_2	498–0	A_2	502–3	A_2	505	$A_1 A_2$	506–9	A_2	$A_1 = 309$		322–323	$A_1 A_2$	326	$A_1 A_2^2$	327	A_2			
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330	$A_1 A_2^2$	331	$A_1 A_2^2$	334–35	A_2	338	$A_1 A_2$	339	$A_1 A_2 A_1 A_2^2$	342	$A_1 A_2 A_1 A_2^3$	343	A_2	346	$A_1 A_2 A_1 A_2^3$	354	$A_1 A_2 A_1 A_2^2$	354	$A_1 A_2$	354	$A_1 A_2$			
355	$A_1 A_2^2$	358–59	A_2	362	$A_1 A_2^2$	367	A_2	370	A_2	378–79	A_2	382	A_2	448–452	$A_1 A_2$	453	A_2	454	$A_1 A_2$	455	A_2			
457–460	$A_1 A_2$	461–63	A_2	465	$A_1 A_2$	467–468	$A_1 A_2$	469	A_2	470	$A_1 A_2$	471	$A_1 A_2^2$	475	$A_1 A_2^2$	476–79	A_2	485–87	A_2	485–87	A_2			
493	A_2	$A_1 = 311$		320	$A_1^3 A_2$	321	$A_1^3 A_2$	322	$A_1^3 A_2$	323	$A_1^2 A_2^2$	324	$A_1^3 A_2$	325	A_2	326	$A_1^2 A_2$	327	A_2	328–330	$A_1^2 A_2$			
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331	$A_1 A_2$	332	$A_1^2 A_2$	333	A_2	334	$A_1 A_2$	335	A_2	336	$A_1^2 A_2$	337	$A_1^2 A_2 A_1 A_2$	338	$A_1^2 A_2 A_1 A_2$	340	$A_1^2 A_2 A_1 A_2$	341	A_2	341	A_2			
342	$A_1 A_2$	343	A_2	344	$A_1^2 A_2$	345–350	$A_1 A_2$	352	$A_1^2 A_2$	353	$A_1^2 A_2^2$	354	$A_1^2 A_2$	355	$A_1 A_2^2$	356	$A_1^2 A_2$	357–59	A_2	360	$A_1^2 A_2$			
361–362	$A_1 A_2$	363	$A_1 A_2^2$	364	$A_1^2 A_2$	365–67	A_2	368	$A_1^2 A_2$	369–370	$A_1 A_2$	371	$A_1 A_2^2$	372	$A_1 A_2$	373–74	A_2	373–74	A_2	373–74	A_2			
376–378	$A_1 A_2$	379	A_2	380–381	$A_1 A_2$	382	A_2	392	$A_1 A_2 A_1 A_2 A_1 A_2^2$	409	$A_1 A_2 A_1 A_2 A_1 A_2^2$	410	$A_1 A_2^2$	411	$A_1 A_2^3$	412	$A_1 A_2 A_1 A_2^2$	413	$A_1 A_2^2$	413	$A_1 A_2^2$			
396–398	$A_1 A_2$	399	A_2	408	$A_1 A_2$	409	$A_1 A_2 A_1 A_2 A_1 A_2^2$	426	$A_1 A_2^2$	427–31	A_2	440	A_2	441	$A_1 A_2^3$	442	A_2	444–46	A_2	448–450	$A_1 A_2^2$			
414–15	A_2	424	$A_1 A_2$	425	$A_1 A_2 A_1 A_2 A_1 A_2^2$	426	$A_1 A_2^2$	427–31	A_2	440	A_2	441	$A_1 A_2^3$	442	A_2	444–46	A_2	448–450	$A_1 A_2^2$	448–450	$A_1 A_2^2$			
451	$A_1 A_2$	452	$A_1^2 A_2$	453	A_2	454	$A_1 A_2$	455	A_2	456	$A_1^2 A_2$	457–460	$A_1 A_2$	461	A_2	462	$A_1 A_2$	463	A_2	491	$A_1 A_2^2$			
464–476	$A_1 A_2$	477	A_2	478	$A_1 A_2$	479	A_2	480–482	$A_1 A_2$	484–87	A_2	483	$A_1 A_2^2$	484–87	A_2	488–490	$A_1 A_2^2$	491	$A_1 A_2^2$	491	$A_1 A_2^2$			
492–93	A_2	496–497	$A_1 A_2$	498–0	A_2	502	A_2	504–505	$A_1 A_2$	506–9	A_2	$A_1 = 319$		320–26	A_1	327	$A_1^3 A_2^2$	328–30	A_1	328–30	A_1			
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331	$A_1^4 A_2$	332–34	A_1	335	A_2	336–38	A_1	339	$A_1^2 A_2$	340–42	A_1	343	$A_1 A_2$	344	A_1	345–346	$A_1^2 A_2$	347	$A_1 A_2$	348	A_1			
349–350	$A_1 A_2$	351	A_2	352–54	A_1	355	$A_1^4 A_2^2$	356–58	A_1	359	A_2	360–62	A_1	363	$A_1^2 A_2^2$	364–66	A_1	367	A_2	368–70	A_1			
371	$A_1 A_2$	372–74	A_1	375	A_2	376	A_1	377–378	$A_1 A_2$	379	$A_1 A_2^2$	380	A_1	381	$A_1 A_2$	382	A_2	448	A_1	460	$A_1^2 A_2$			
449–450	$A_1^3 A_2$	451	$A_1^2 A_2$	452	A_1	453–454	$A_1^2 A_2$	455	$A_1 A_2$	457–458	$A_1 A_2$	469–471	$A_1 A_2$	475–478	$A_1 A_2$	479	A_2	484	A_1	484	A_1			
461–462	$A_1 A_2$	463	A_2	465	$A_1^2 A_2$	467	$A_1 A_2$	468	$A_1^2 A_2$	469–471	$A_1 A_2$	475–478	$A_1 A_2$	479	A_2	484	A_1	484	A_1	484	A_1			
485	A_2	486	$A_1 A_2$	487	A_2	493–94	A_2	501	A_2	$A_1 = 351$		381	A_1	438–39	A_1	446	A_1	447	A_2	484	A_1			
381	$A_1 A_2$	415	A_2	447	A_2	$A_1 = 375$		376–78	A_1	379	$A_1 A_2$	380	A_1	381	$A_1^2 A_2$	382	A_1	384–42	A_1	444–45	A_1	446–47	A_2	
448–62	A_1	463	A_2	464–70	A_1	471	$A_1^2 A_2$	472–74	A_1	475	$A_1 A_2$	476	A_1	477–478	$A_1 A_2$	479	A_2	480–86	A_1	487	A_2	487	A_2	
488–90	A_1	491	$A_1 A_2$	492	A_1	493	A_2	496–98	A_1	499	$A_1 A_2$	500	A_1	502	A_2	504	A_1	505	$A_1 A_2$	506–9	A_2	506–9	A_2	
387	$A_1^3 A_2$	391	$A_1^2 A_2$	395	$A_1^2 A_2$	399	$A_1 A_2$	403	$A_1^2 A_2$	407	$A_1 A_2$	411	$A_1 A_2$	415	$A_1 A_2$	423	$A_1 A_2$	427	$A_1^2 A_2$	431	$A_1 A_2$	439	$A_1 A_2$	
443	A_2	447	A_2	455	$A_1 A_2$	463	$A_1 A_2$	471	$A_1 A_2$	479	A_2	$A_1 = 383$		384–78	A_1	479	$A_1 A_2$	480–94	A_1	496–2	A_1	503	$A_1 A_2$	
504–6	A_1	507	$A_1 A_2$	508–9	A_1	<hr/>																		

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